Recent results on multiterminal Josephson junctions

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List of Collaborators: theory

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Benoît Douçot

Mathematics Department of INSA (Rouen, France)

Jean-Guy Caputo

List of Collaborators: experiments

Karlsruhe Institute of Technology (KIT, Germany)

Group of **Romain Danneau**

Grenoble (Néel and CEA)

Groups of **Hervé Courtois** and **François Lefloch**

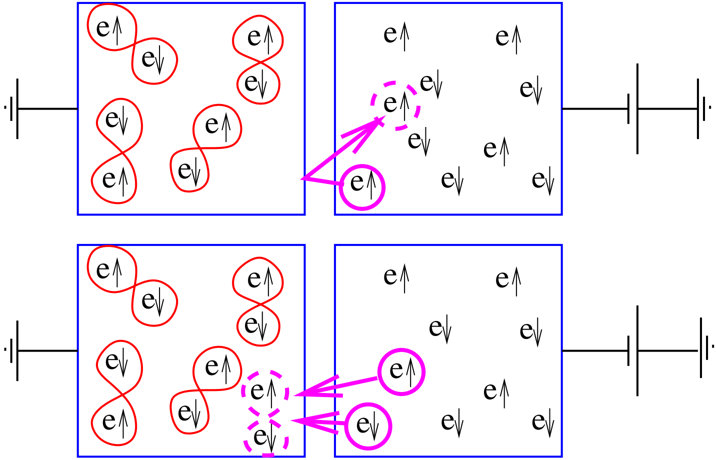
Weizmann Institute (Rehovot, Israel)

Part of group of **Moty Heiblum**:
Yuval Ronen, Yonatan Cohen, Jung-Hyun Kang, Hadas Shtrikman

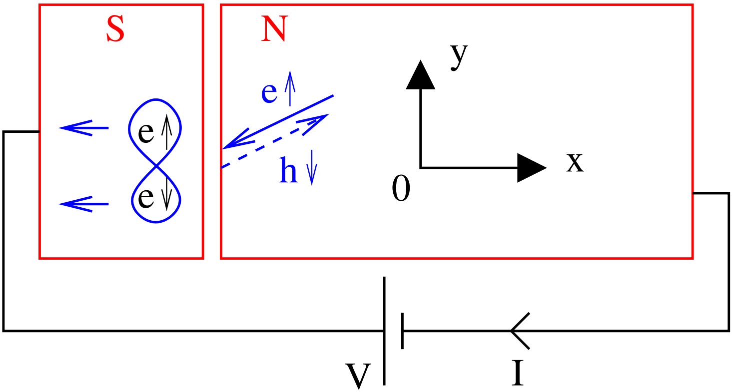
Harvard University

Part of group of **Philip Kim**:
Katie Huang, Yuval Ronen

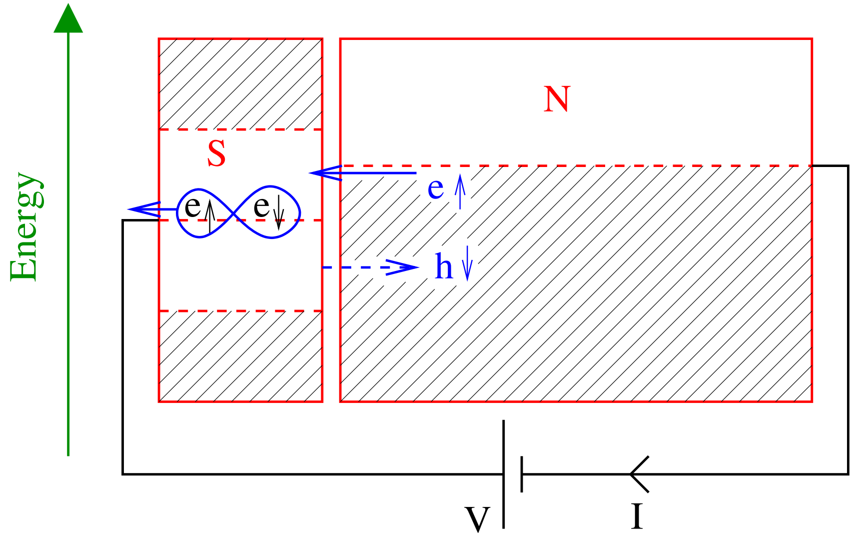
SN Junction: Andreev Reflection (1/3)



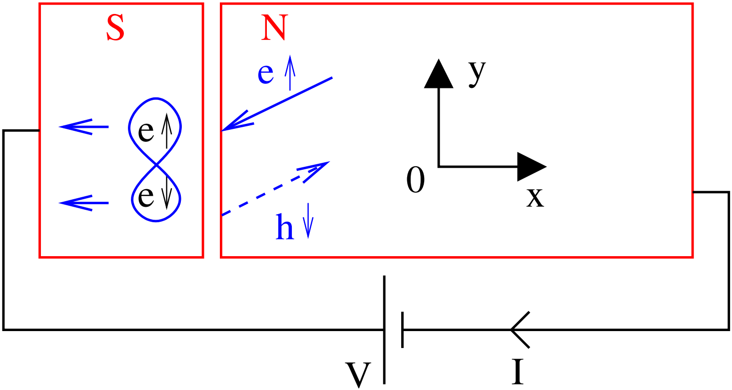
SN Junction: Andreev Reflection (2/3)

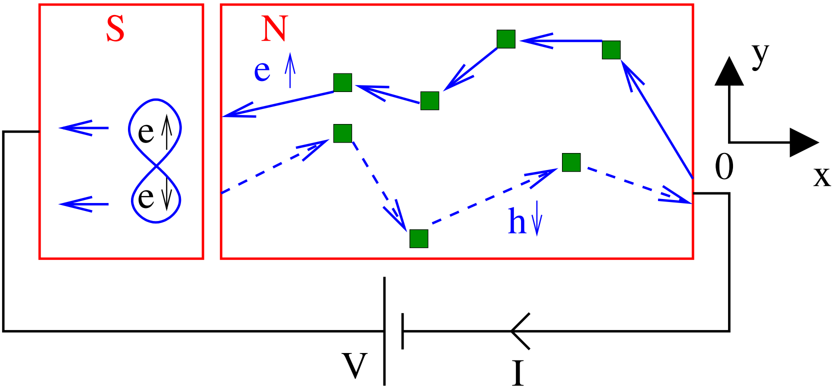


SN Junction: Andreev Reflection (3/3)



SN Junction: Nonlocal Andreev Reflection (1/3)

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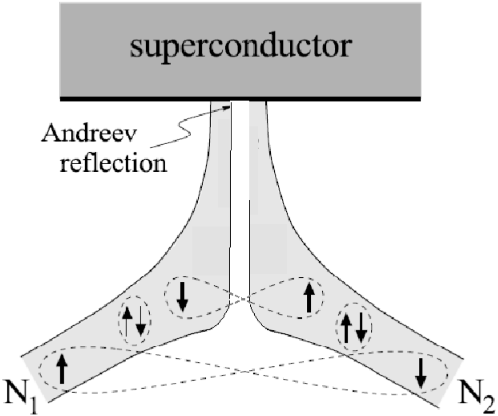
Nonlocal Andreev Reflection ≡ Cooper Pair Splitting (3/3)

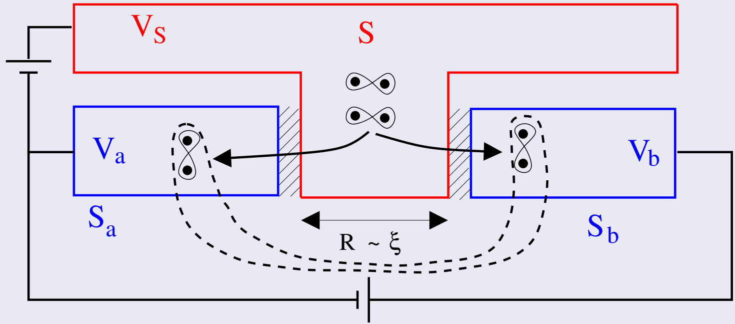


Three-terminal set-up required in experiments

First theoretical contributions: Byers-Flatté, Martin, Anatram-Datta, Deutscher-Feinberg, Falci-Hekking, Choi-Bruder-Loss, Mélin

Production of Nonlocal Quartets, $R < \xi$

A. Freyn, B. Douçot, D. Feinberg, R. Mélin, PRL 2011, NÉEL / LPTHE

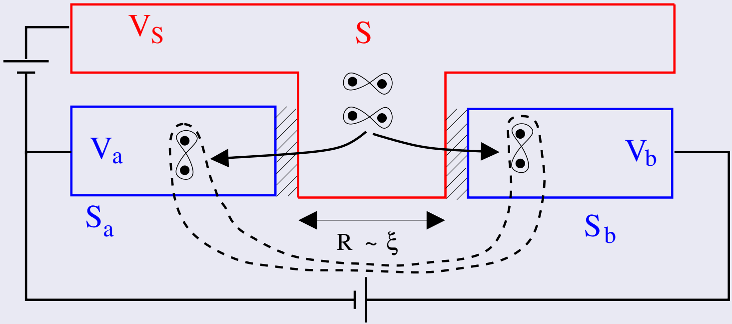


Intuitively

2 pairs in coherence volume within time interval $\hbar/\Delta$
$\rightarrow$ production of a correlated pair of pair between S_a and S_b

Production of Nonlocal Quartets, $R < \xi$

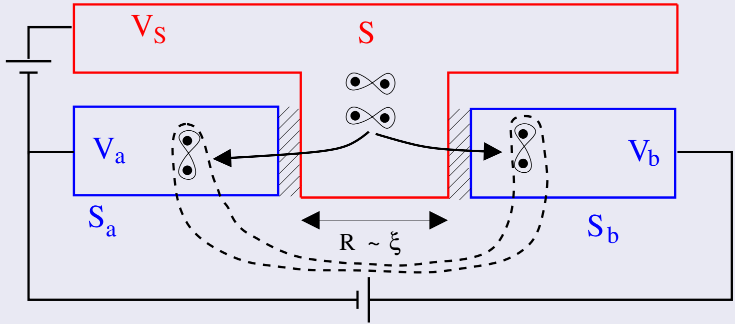
A. Freyn, B. Douçot, D. Feinberg, R. Mélin, PRL 2011, NÉEL / LPTHE



ac Josephson current of pairs from S_a to S and from S to S_b

Production of Nonlocal Quartets, $R < \xi$

A. Freyn, B. Douçot, D. Feinberg, R. Mélin, PRL 2011, NÉEL / LPTHE

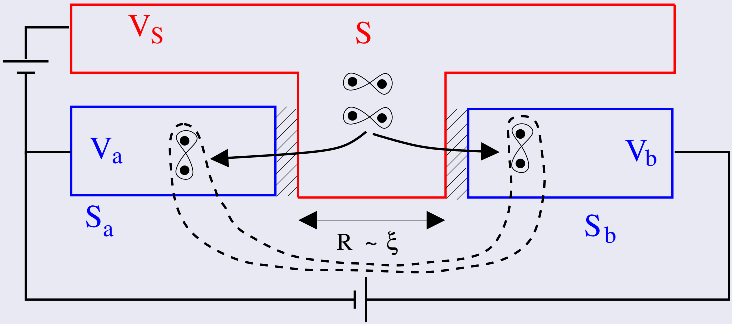


ac Josephson current of pairs from S_a to S and from S to S_b

But possibility of a dc Josephson current of quartets from S to S_a and S_b if $V_a = -V_b$ and $V_S = 0$ because $\Delta E = 2e(V_a + V_b - 2V_S) \equiv 0$

Production of Nonlocal Quartets, $R < \xi$

A. Freyn, B. Douçot, D. Feinberg, R. Mélin, PRL 2011, NÉEL / LPTHE



Adiabatic model

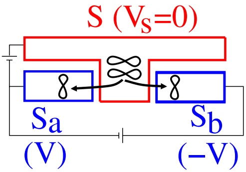
$$\phi_a(t) + \phi_b(t) - 2\phi_S = [2e(V_a + V_b - 2V_S)/\hbar]t + \phi_a + \phi_b - 2\phi_S$$
$$I_{quartet}(t) = I_c \sin[\phi_a(t) + \phi_b(t) - 2\phi_S]$$

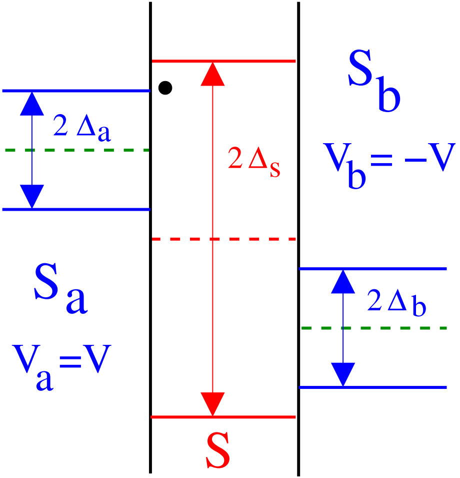
- AC Josephson effect of quartets in general
- DC Josephson effect of quartets if $V_a = -V_b$ and $V_S = 0$

Nonlocal Quartets, DC if $V_a = -V_b$, $V_S = 0$

Quartet Resonance in DC Current for $V_a = -V_b$

A. Freyn, B. Douçot, D. Feinberg, R. Mélin, PRL 2011, NÉEL / LPTHE





Nonlocal Quartets, DC if $V_a = -V_b$, $V_S = 0$

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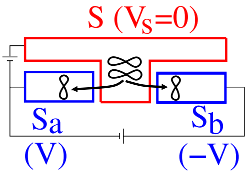

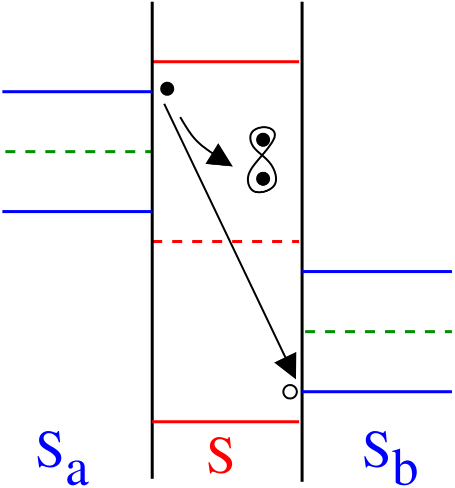

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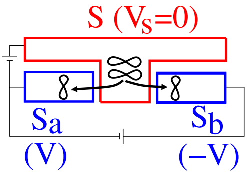

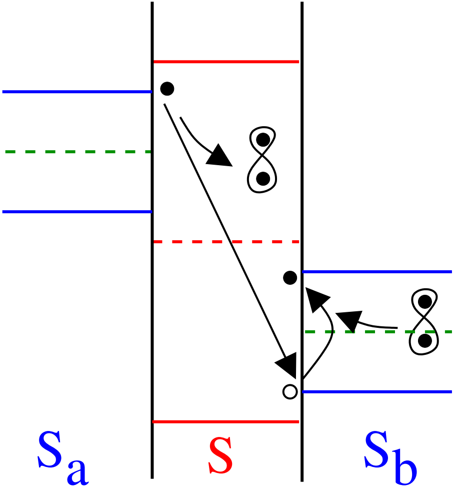

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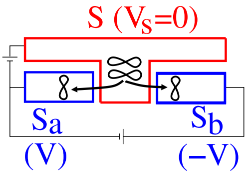

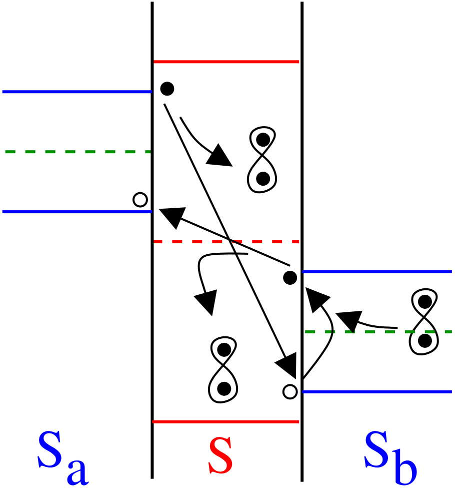

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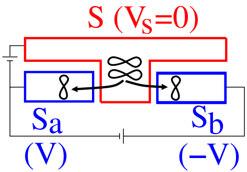

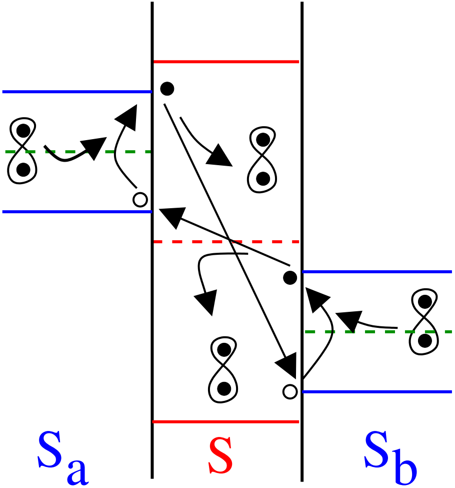

Nonlocal Quartets, DC if $V_a = -V_b$, $V_S = 0$

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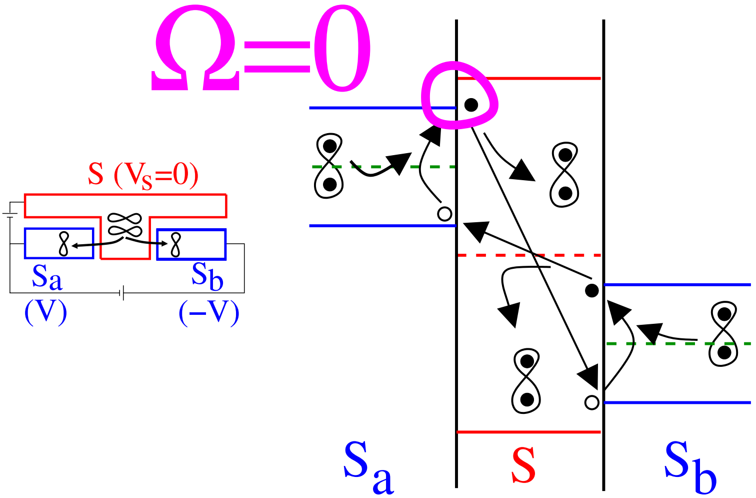
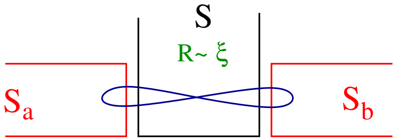


π-junction for the electron quartets

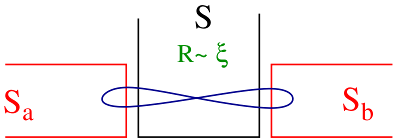


Split pair

$$\frac{1}{\sqrt{2}}\left(c^+_{a,\uparrow}c^+_{b,\downarrow} - c^+_{a,\downarrow}c^+_{b,\uparrow}\right)$$

Unstable

π-junction for the electron quartets



Split pair

$$\frac{1}{\sqrt{2}}\left(c^+_{a,\uparrow}c^+_{b,\downarrow}-c^+_{a,\downarrow}c^+_{b,\uparrow}\right)$$

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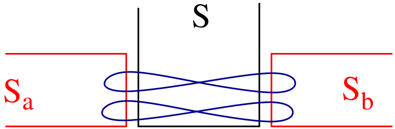
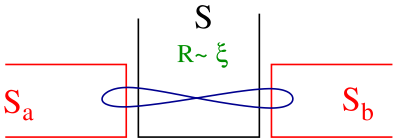


$$\frac{1}{2}\left(c^+_{a,\uparrow}c^+_{b,\downarrow}-c^+_{a,\downarrow}c^+_{b,\uparrow}\right)^2$$

π-junction for the electron quartets



Split pair

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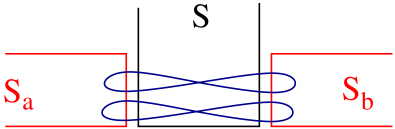
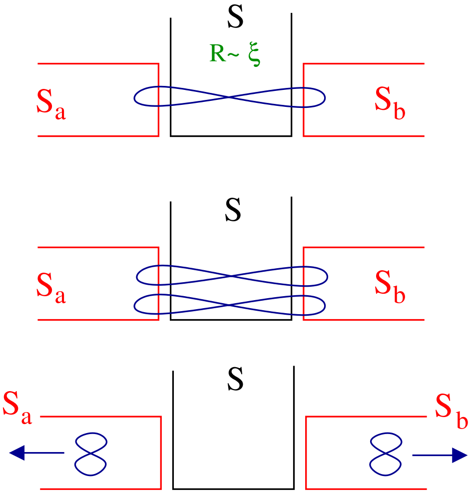


$$\frac{1}{2}\left(c^+_{a,\uparrow}c^+_{b,\downarrow}-c^+_{a,\downarrow}c^+_{b,\uparrow}\right)^2$$

$$=$$

$$-\left(c^+_{a,\uparrow}c^+_{a,\downarrow}\right)\left(c^+_{b,\uparrow}c^+_{b,\downarrow}\right)$$

π-junction for the electron quartets



Split pair

$$\frac{1}{\sqrt{2}}\left(c^+_{a,\uparrow}c^+_{b,\downarrow} - c^+_{a,\downarrow}c^+_{b,\uparrow}\right)$$

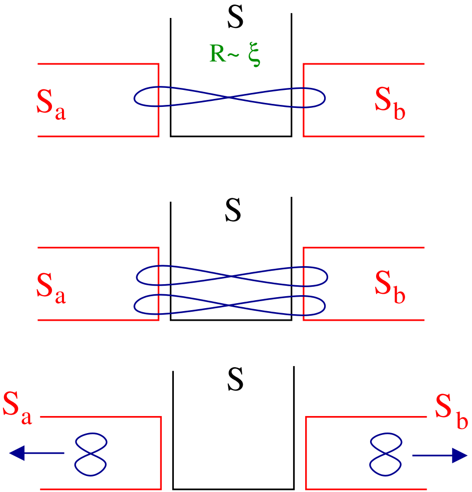
Unstable

$$\frac{1}{2}\left(c^+_{a,\uparrow}c^+_{b,\downarrow} - c^+_{a,\downarrow}c^+_{b,\uparrow}\right)^2$$

$$=$$

$$-\left(c^+_{a,\uparrow}c^+_{a,\downarrow}\right)\left(c^+_{b,\uparrow}c^+_{b,\downarrow}\right)$$

π-junction for the electron quartets



$$\frac{1}{2}\left(c^+_{a,\uparrow}c^+_{b,\downarrow} - c^+_{a,\downarrow}c^+_{b,\uparrow}\right)^2 = -\left(c^+_{a,\uparrow}c^+_{a,\downarrow}\right)\left(c^+_{b,\uparrow}c^+_{b,\downarrow}\right)$$

No preformed quartets

Glue between pairs $=$ interfaces with $R \sim \xi$

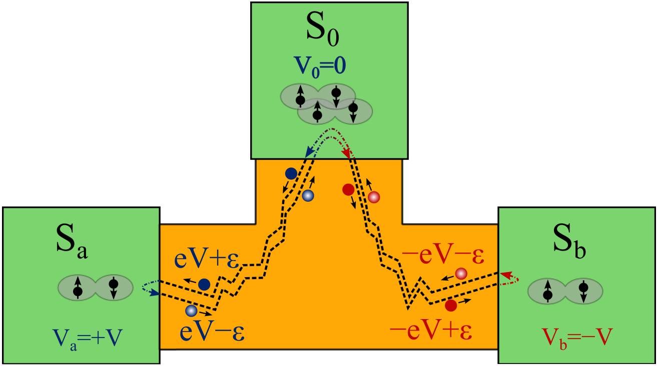
$-$ sign $\Rightarrow$ π-junction

$$I_{quartet} = -|I_c|\sin(\varphi_a + \varphi_b - 2\varphi_S)$$

Macroscopic manifestation of the internal structure of a pair (orbital and spin symmetries)

Experiment on Quartets in a Metallic Cu-Al Structure (Grenoble group)

A.H. Pfeffer, J.E. Duvauchelle, H. Courtois, R. Mélin, D. Feinberg and F. Lefloch, PRB '14



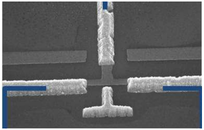
Theoretical calculation

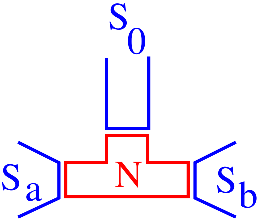
- Perturbative expansion in the tunnel amplitudes

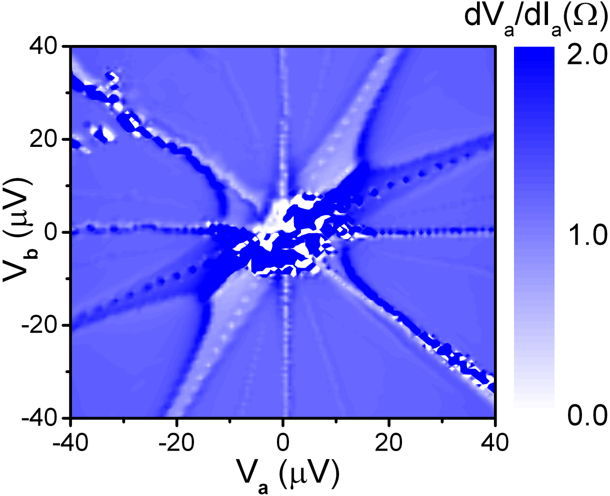
$\Rightarrow$ Diffusion modes, evaluated in the ladder approximation

Synchronization for a trijunction ($T = 200$ mK)

A.H. Pfeffer, J.E. Duvauchelle, H. Courtois, F. Lefloch, R. Mélin, D. Feinberg, PRB'14







Synchronization for a trijunction

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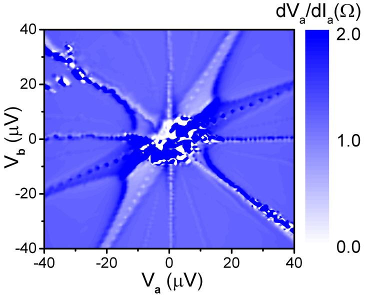
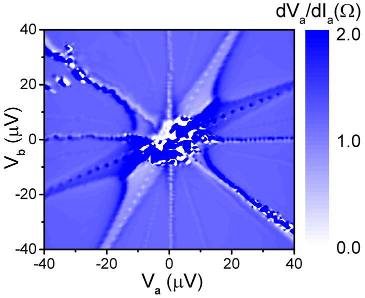


Direct Josephson $S_a - S_0$ and $S_b - S_0$

Two resonances for $V_a = 0$ and $V_b = 0$

Synchronization for a trijunction

A.H. Pfeffer, J.E. Duvauchelle, H. Courtois, F. Lefloch, R. Mélin, D. Feinberg, PRB'14

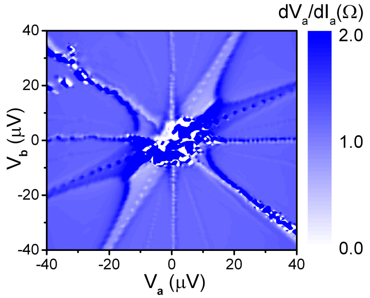


Direct Josephson $S_a - S_b$

- Resonance expected at $V_a = V_b$
- However, not visible because lock-in excitation on S_0

Synchronization for a trijunction

A.H. Pfeffer, J.E. Duvauchelle, H. Courtois, F. Lefloch, R. Mélin, D. Feinberg, PRB'14

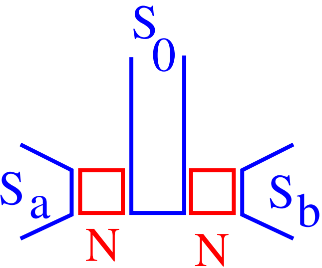


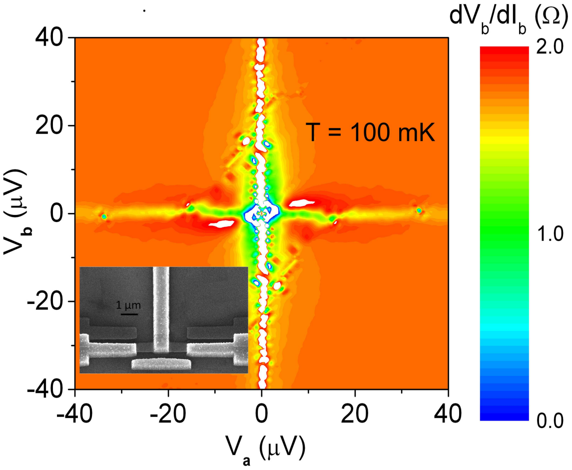
Three additional resonance lines

- $2V_0 = V_a + V_b$; $2V_a = V_0 + V_b$; $2V_b = V_0 + V_a$
- Just permutation of the 3 terminals $\rightarrow$ equivalent resonances
- Are they due to quartets or to classical synchronization by an external impedance ?

No Resonances for a Separated Bijunction
A.H. Pfeffer, J.E. Duvauchelle, H. Courtois, R. Mélin, D. Feinberg, F. Lefloch, PRB '14

2 resonances for direct Josephson $S_0 - S_a$ and $S_0 - S_b$





PHYSICAL REVIEW B **93**, 115436 (2016)

Gate-tunable zero-frequency current cross correlations of the quartet state in a voltage-biased three-terminal Josephson junction

Régis Mélin, Moïse Sotto, and Denis Feinberg
Université Grenoble-Alpes, Institut Néel, BP 166, F-38042 Grenoble Cedex 9, France
and CNRS, Institut Néel, BP 166, F-38042 Grenoble Cedex 9, France

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Benoît Douçot
Laboratoire de Physique Théorique et des Hautes Energies, CNRS UMR 7589, Université Pierre et Marie Curie, Sorbonne Universités, 4 Place Jussieu, 75252 Paris Cedex 05

(Received 26 November 2015; revised manuscript received 3 March 2016; published 25 March 2016)

A three-terminal Josephson junction biased at opposite voltages can sustain a phase-sensitive dc current carrying three-body static phase coherence, known as the "quartet current." We calculate the zero-frequency current noise cross correlations and answer the question of whether this current is noisy (like a normal current in response to a voltage drop) or noiseless (like an equilibrium supercurrent in response to a phase drop). A quantum dot with a level at energy ϵ_0 is connected to three superconductors S_a, S_b, and S_c with gap Δ, biased at $V_a = V$,

PNAS

Nonlocal supercurrent of quartets in a three-terminal Josephson junction

Yonatan Cohen[a,1], Yuval Ronen[a,1], Jung-Hyun Kang[a], Moty Heiblum[a,2], Denis Feinberg[b], Régis Mélin[b], and Hadas Shtrikman[a]

[a]Department of Condensed Matter Physics, Braun Center for Submicron Research, Weizmann Institute of Science, 76100 Rehovot, Israel; and [b]Institut Néel, CNRS, Université Grenoble-Alpes, Institute of Engineering (INP), 38000 Grenoble, France

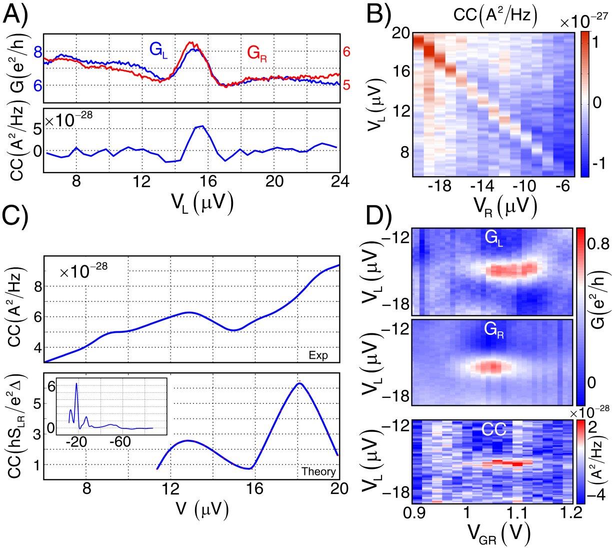
Edited by Eduardo Fradkin, University of Illinois at Urbana–Champaign, Urbana, IL, and approved May 21, 2018 (received for review January 2, 2018)

A novel nonlocal supercurrent, carried by quartets, each consisting of four electrons, is expected to appear in a voltage-biased three-terminal Josephson junction. This supercurrent results from a non-

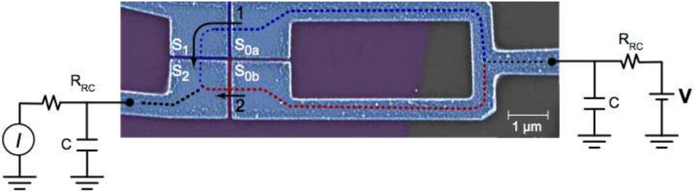
several alternative models for that current could not be ruled out. Here, we verify an emergent coherent quartet supercurrent in a 3TJ, which is formed in a proximitized semiconducting

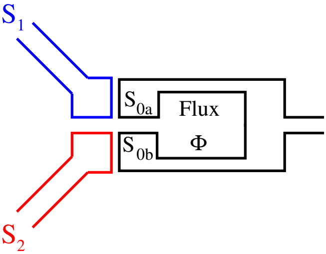
Zero-Frequency Cross-Correlations

Y. Cohen, Y. Ronen, J.-H. Kang, M. Heiblum,
D. Feinberg, R. Mélin and H. Shtrikman, PNAS 2018



K.-F. Huang, Y. Ronen, R. Mélin, D. Feinberg, K. Watanabe T. Taniguchi and P. Kim, Nature Comm. '22





The quartet resonance line (1/2)

K.-F. Huang, Y. Ronen, R. Mélin, D. Feinberg, K. Watanabe, T. Taniguchi and P. Kim, Nature Comm. 2022

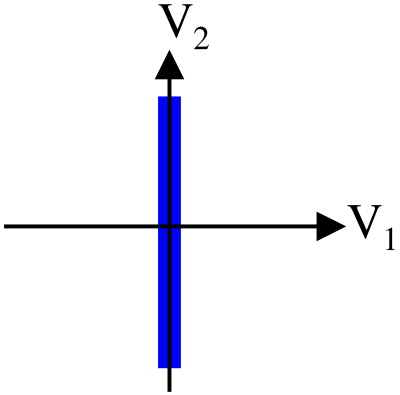
A. Freyn, B. Douçot, D. Feinberg and R. Mélin, Phys. Rev. Lett. 106, 257005 (2011)

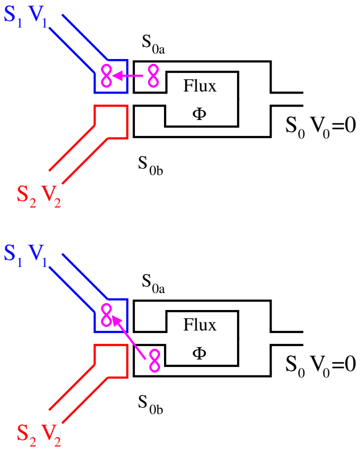
First, the two-terminal processes

The quartet resonance line (1/2)

K.-F. Huang, Y. Ronen, R. Mélin, D. Feinberg, K. Watanabe, T. Taniguchi and P. Kim, Nature Comm. 2022
A. Freyn, B. Douçot, D. Feinberg and R. Mélin, Phys. Rev. Lett. 106, 257005 (2011)

DC-Josephson effect between S_1 and $S_0 \Rightarrow V_1 = 0$





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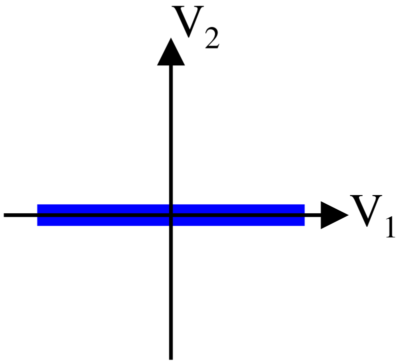
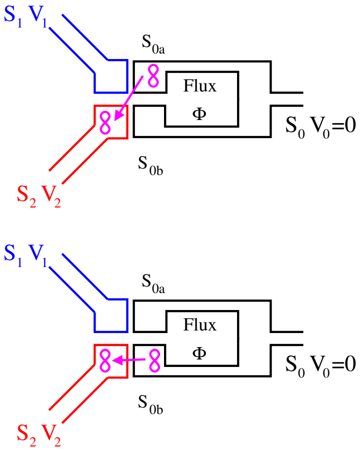
K.-F. Huang, Y. Ronen, R. Mélin, D. Feinberg, K. Watanabe, T. Taniguchi and P. Kim, Nature Comm. 2022

A. Freyn, B. Douçot, D. Feinberg and R. Mélin, Phys. Rev. Lett. 106, 257005 (2011)

DC-Josephson effect between S_2 and $S_0 \Rightarrow V_2 = 0$

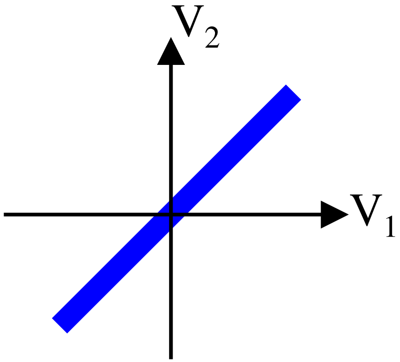


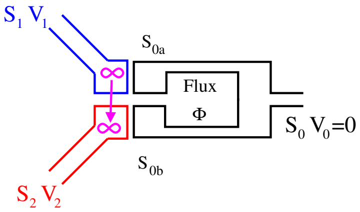


The quartet resonance line (1/2)

K.-F. Huang, Y. Ronen, R. Mélin, D. Feinberg, K. Watanabe, T. Taniguchi and P. Kim, Nature Comm. 2022
A. Freyn, B. Douçot, D. Feinberg and R. Mélin, Phys. Rev. Lett. 106, 257005 (2011)

DC-Josephson effect between S_1 and $S_2 \Rightarrow V_1 = V_2$





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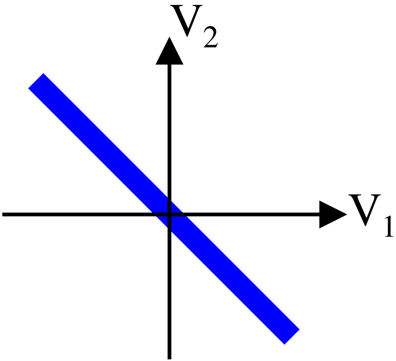
A. Freyn, B. Douçot, D. Feinberg and R. Mélin, Phys. Rev. Lett. 106, 257005 (2011)

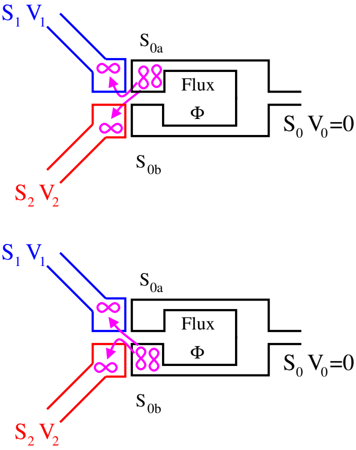
Second, the three-terminal processes

The quartet resonance line (1/2)

K.-F. Huang, Y. Ronen, R. Mélin, D. Feinberg, K. Watanabe, T. Taniguchi and P. Kim, Nature Comm. 2022
A. Freyn, B. Douçot, D. Feinberg and R. Mélin, Phys. Rev. Lett. 106, 257005 (2011)

Quartet from $S_{0,a}$ or $S_{0,b}$ towards $(S_1, S_2) \Rightarrow V_1 + V_2 = 0$





The quartet resonance line (1/2)

K.-F. Huang, Y. Ronen, R. Mélin, D. Feinberg, K. Watanabe, T. Taniguchi and P. Kim, Nature Comm. 2022

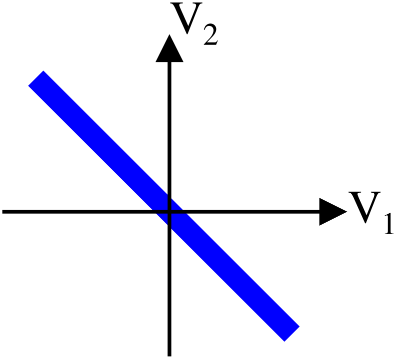
A. Freyn, B. Douçot, D. Feinberg and R. Mélin, Phys. Rev. Lett. 106, 257005 (2011)

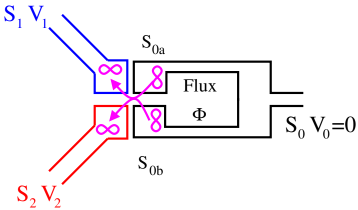
Third, the four-terminal processes

The quartet resonance line (1/2)

K.-F. Huang, Y. Ronen, R. Mélin, D. Feinberg, K. Watanabe, T. Taniguchi and P. Kim, Nature Comm. 2022
A. Freyn, B. Douçot, D. Feinberg and R. Mélin, Phys. Rev. Lett. 106, 257005 (2011)

Split quartet from $(S_{0,a},\, S_{0,b})$ towards $(S_1,\, S_2) \Rightarrow V_1 + V_2 = 0$

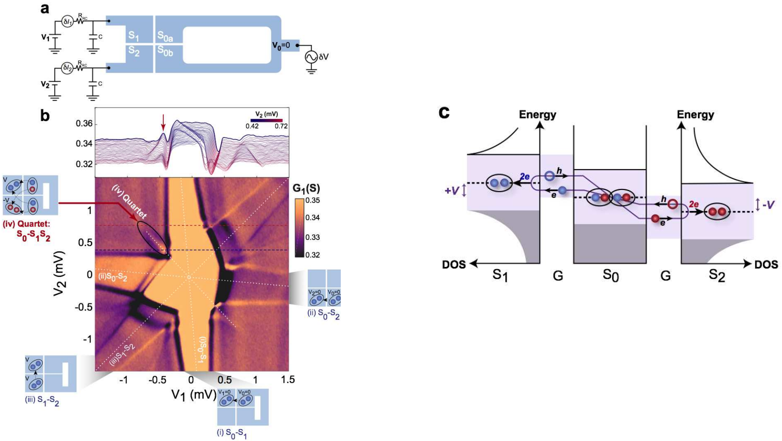




The quartet resonance line (2/2)

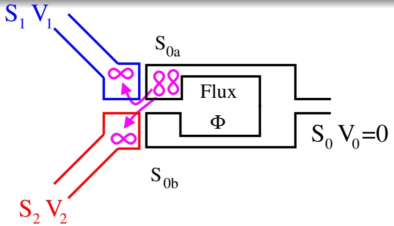
K.-F. Huang, Y. Ronen, R. Mélin, D. Feinberg, K. Watanabe, T. Taniguchi and P. Kim, Nature Comm. 2022

A. Freyn, B. Douçot, D. Feinberg and R. Mélin, Phys. Rev. Lett. 106, 257005 (2011)

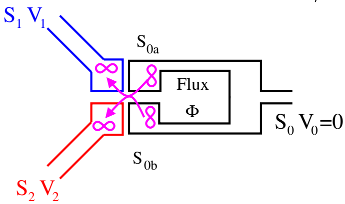


The flux periodicity (1/2)

K.-F. Huang, Y. Ronen, R. Mélin, D. Feinberg, K. Watanabe, T. Taniguchi and P. Kim, Nature Comm. 2022
R. Mélin, Phys. Rev. B **102**, 245435 (2020)



Interference between $3TQ_a$, $3TQ_b$ $\Rightarrow$ $h/4e$-periodicity



Interference between $3TQ_a$, $3TQ_b$, $4TSQ$ $\Rightarrow$ $h/2e$-periodicity

The flux periodicity (2/2)

K.-F. Huang, Y. Ronen, R. Mélin, D. Feinberg, K. Watanabe, T. Taniguchi and P. Kim, Nature Comm. 2022

In experiments, balance between the $3TQ$ and the $4TSQ$ tuned by gate voltage.

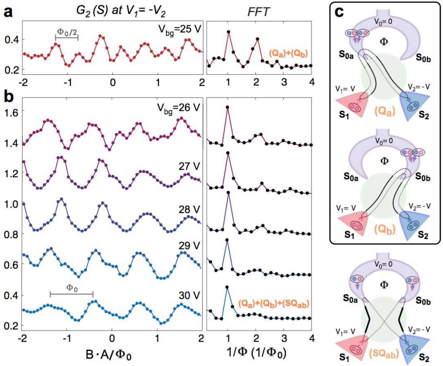
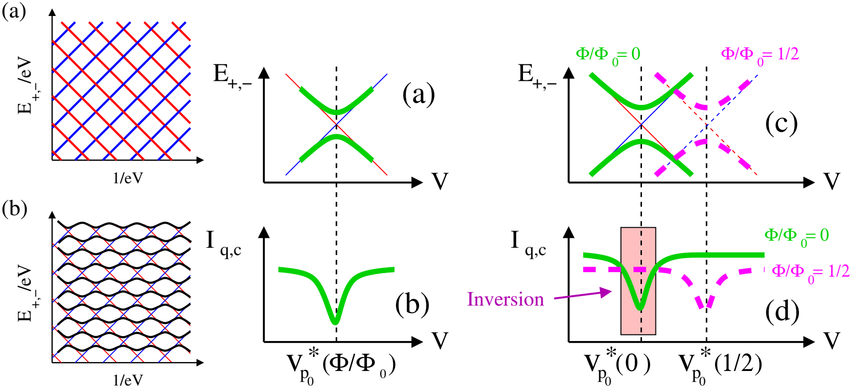


Figure 3. Different types of quartet process. a, Left panel shows the quartet differential conductance G_2 $(= \delta I_2/\delta V)$ measured at S_2 a function of the flux $\Phi = B \cdot A/\Phi_0$ for V_{bg} = 25 V. Satisfying the quartet bias condition, $V_1 = -V_2$ is fixed at 4 V. Right panel shows the analysis from fast Fourier transform (FFT) with a prominent second harmonic. **b**, for V_{bg} = 26-30 V. The periodicity evolves from half-flux quantum to one flux quantum as V_{bg} increases. **c**, Subfigure (Qa)(Qb) shows the conventional three-terminal quartet process with only one out of the two loop contacts involved. Electron-hole conversion happens twice at the same contact of the loop (either S_{0a} or S_{0b}), resulting in periodicity of half-flux quantum. Subfigure (SQab) shows the split-quartet process involving both contacts of the loop. With the odd parity of Cooper pairs transferred, the periodicity is one flux quantum.

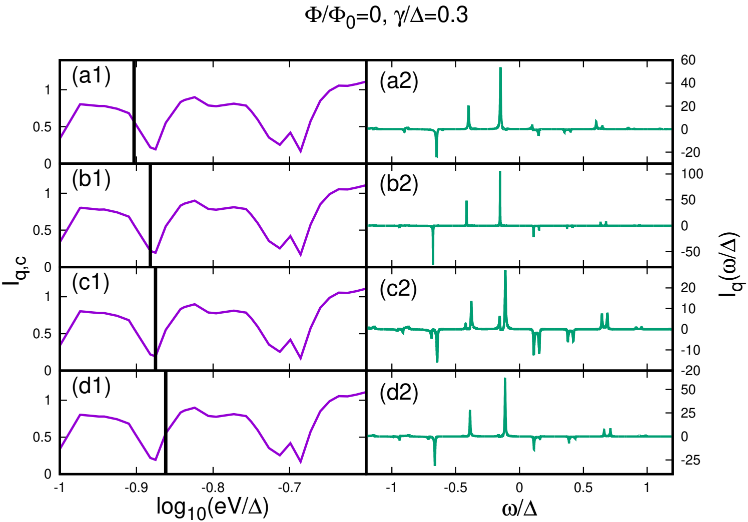
Floquet resonances and the voltage dependence
R. Mélin and B. Douçot, Phys. Rev. B **102**, 245436 (2020)

0D quantum dots. Floquet replica of the ABS spectrum.



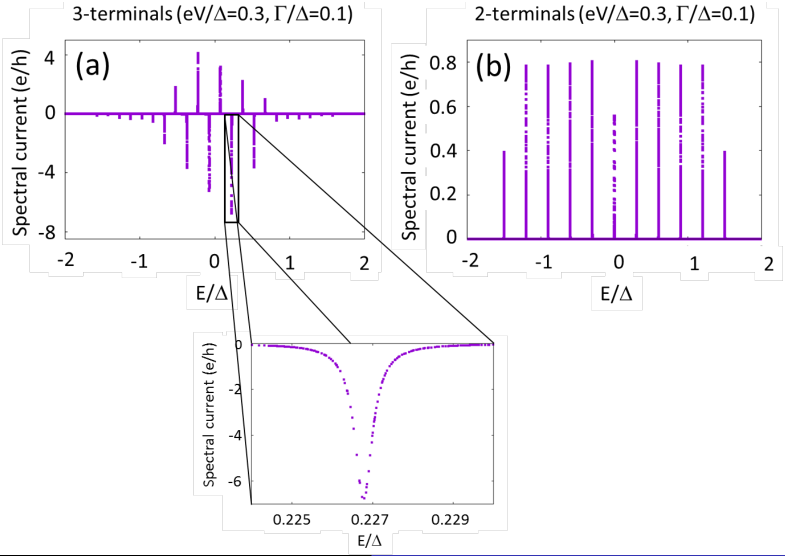
Hybridization between the two Floquet ladders

R. Mélin and B. Douçot, Phys. Rev. B **102**, 245436 (2020)



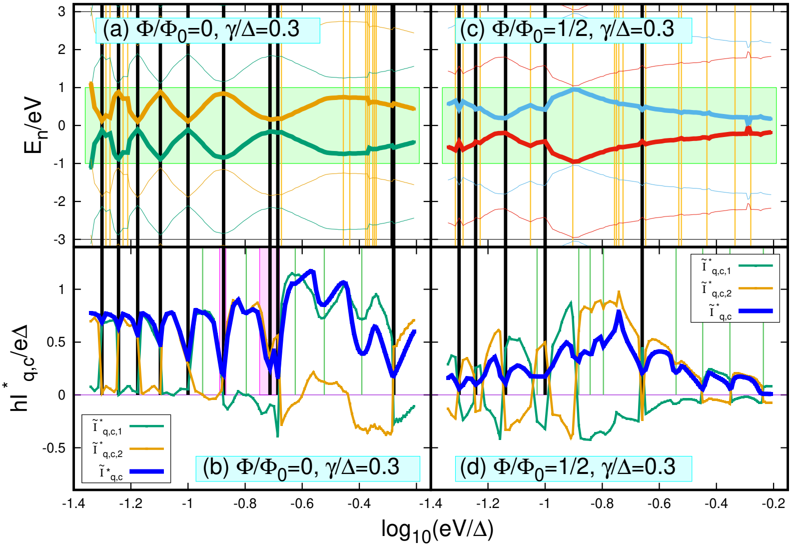
Spectral Current: Evidence for two Floquet-Wannier-Stark Ladders

R. Mélin, J.-G. Caputo, K. Yang and B. Douçot, PRB '17



Quartet critical current-voltage characteristics

R. Mélin and B. Douçot, Phys. Rev. B **102**, 245436 (2020)



Conclusions (1/2)

Previous experiments were shown to be compatible with the quartets:

- The Grenoble group experiment:
  A. H. Pfeffer, J. E. Duvauchelle, H. Courtois, R. Mélin, D. Feinberg, and F. Lefloch,
  Phys. Rev. B **90**, 075401 (2014).
- The Weizmann group experiment:
  Y. Cohen, Y. Ronen, J.-H. Kang, M. Heiblum, D. Feinberg, R. Mélin, and H. Shtrikman,
  PNAS July **3**, 2018 115 (27) 6991-6994.

Compatibility between new Harvard group experiment and theory of four-terminal quartets.

Conclusions (2/2)

- Interference between quartet-ABS produces cross-over between $h/4e$- and $h/2e$-periodicity.
- New channel of four-terminal split quartets produces $I_{q,c}(\Phi = \pi, V) \neq I_{q,c}(\Phi = 0, V)$.

Two mechanisms for Floquet theory:

- Without relaxation: dips in the quartet critical current-voltage characteristics due to quantum coherent superpositions of the time-dependent ABS.
- With relaxation: Resonance peaks are inverse proportional to the Floquet line-width.