NONLINEAR WAVES AND NETWORKS (ONL 2022)

NONLINEAR WAVES IN Y-JUNCTIONS

Presented by Dr. Denys DUTYKH (CNRS / Univ. Savoie Mont Blanc)

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THE GENESIS:

PRL **97,** 017004 (2006) PHYSICAL REVIEW LETTERS week ending 7 JULY 2006

Flux Cloning in Josephson Transmission Lines

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(Received 30 March 2006; published 7 July 2006)

We describe a novel effect related to the controlled birth of a single Josephson vortex. In this phenomenon, the vortex is created in a Josephson transmission line at a T-shaped junction. The "baby" vortex arises at the moment when a "mother" vortex propagating in the adjacent transmission line passes the T-shaped junction. In order to give birth to a new vortex, the mother vortex must have enough kinetic energy. Its motion can also be supported by an externally applied driving current. We determine the critical velocity and the critical driving current for the creation of the baby vortices and briefly discuss the potential applications of the found effect.

DOI: 10.1103/PhysRevLett.97.017004 PACS numbers: 85.25.Cp, 03.75.Lm, 05.45.Yv, 74.50.+r

The main finding:

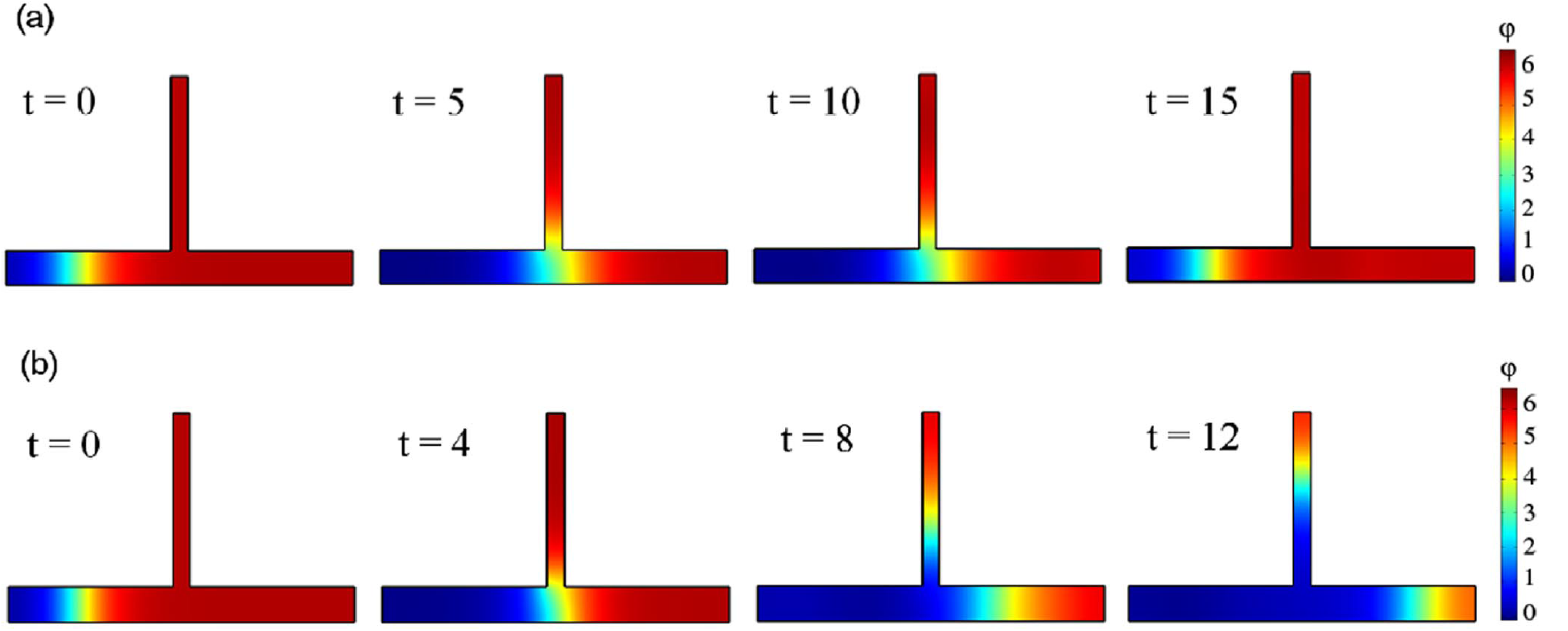

FIG. 2 (color online). (a) Reflection of an incident fluxon propagating with velocity $u = 0.7$ and (b) cloning of a fluxon propagating with velocity 0.8 higher than the critical $u_c = 0.76$ (normalized to the Swihart velocity $\bar{c}$). Both diagrams represent numerical simulations of the superconducting phase difference for the geometry in Fig. 1(a) with the use of the 2D sine-Gordon equation. The driving current is absent. The color scale represents the superconducting phase difference φ.

Our (first) work with JGC:

PHYSICAL REVIEW E **90**, 022912 (2014)

Nonlinear waves in networks: Model reduction for the sine-Gordon equation

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(Received 26 February 2014; revised manuscript received 18 June 2014; published 25 August 2014)

To study how nonlinear waves propagate across Y- and T-type junctions, we consider the two-dimensional (2D) sine-Gordon equation as a model and examine the crossing of kinks and breathers. Comparing energies for different geometries reveals that, for small widths, the angle of the fork plays no role. Motivated by this, we introduce a one-dimensional effective model whose solutions agree well with the 2D simulations for kink and breather solutions. These exhibit two different behaviors: a kink crosses if it has sufficient energy; conversely a breather crosses when $v > 1 - \omega$, where v and ω are, respectively, its velocity and frequency. This methodology can be generalized to more complex nonlinear wave models.

DOI: 10.1103/PhysRevE.90.022912 PACS number(s): 05.45.Yv, 74.81.Fa

The target geometry:

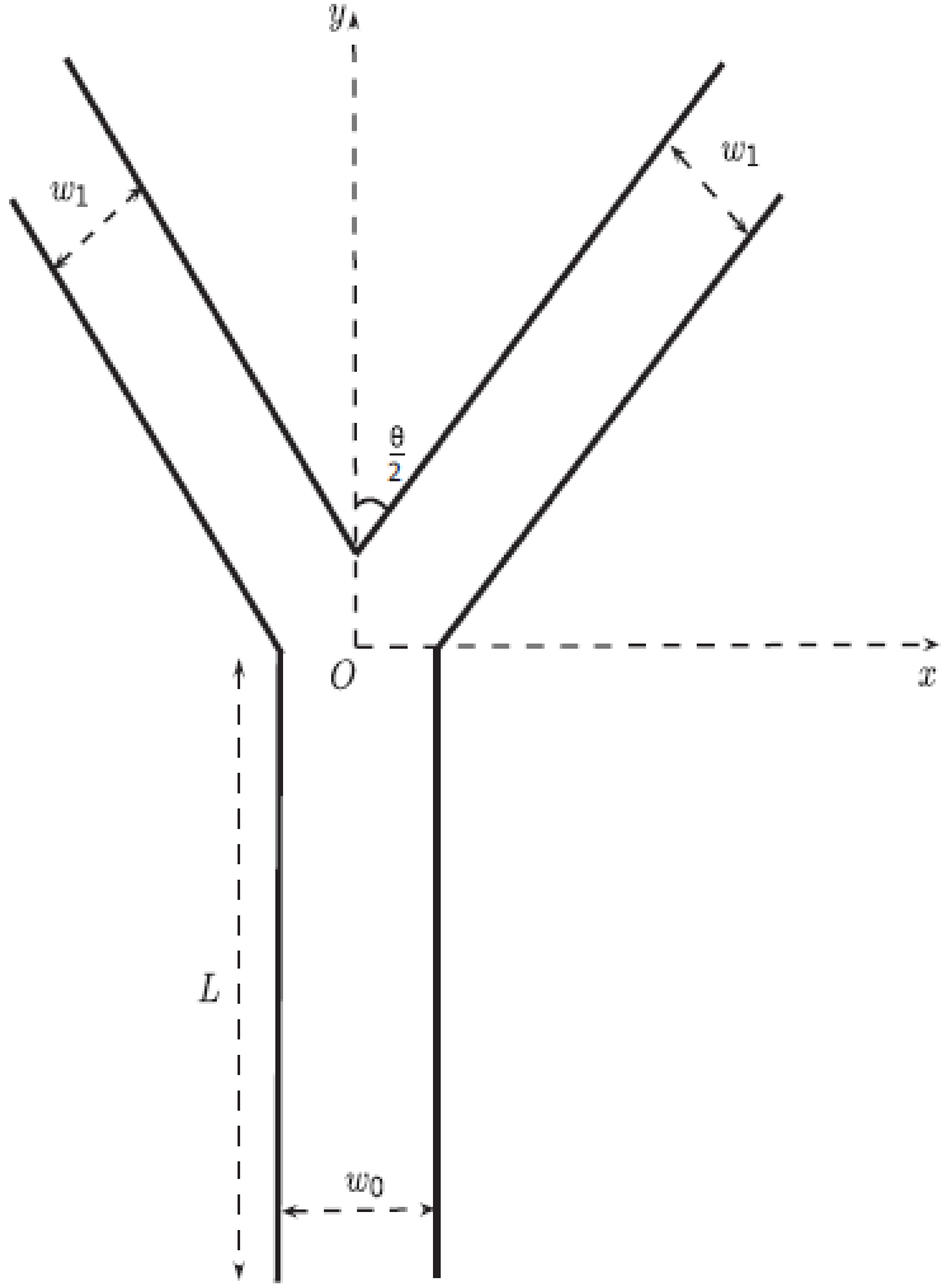


FIG. 1. Sketch of the computational domain Ω.

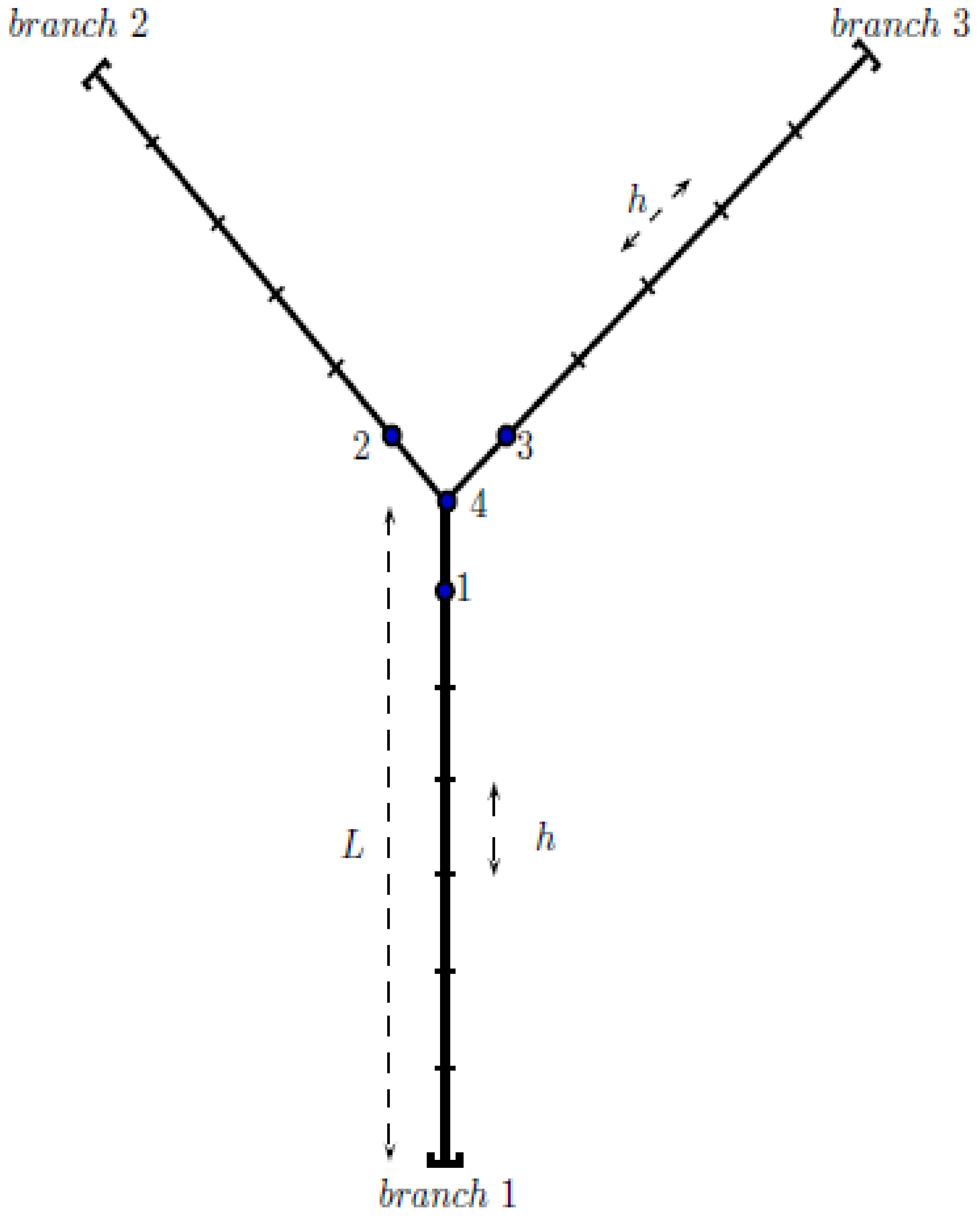


FIG. 2. (Color online) Sketch of the tree geometry for the 1D effective model.

GOVERNING EQUATIONS (2D):

We consider the 2D sine-Gordon equation

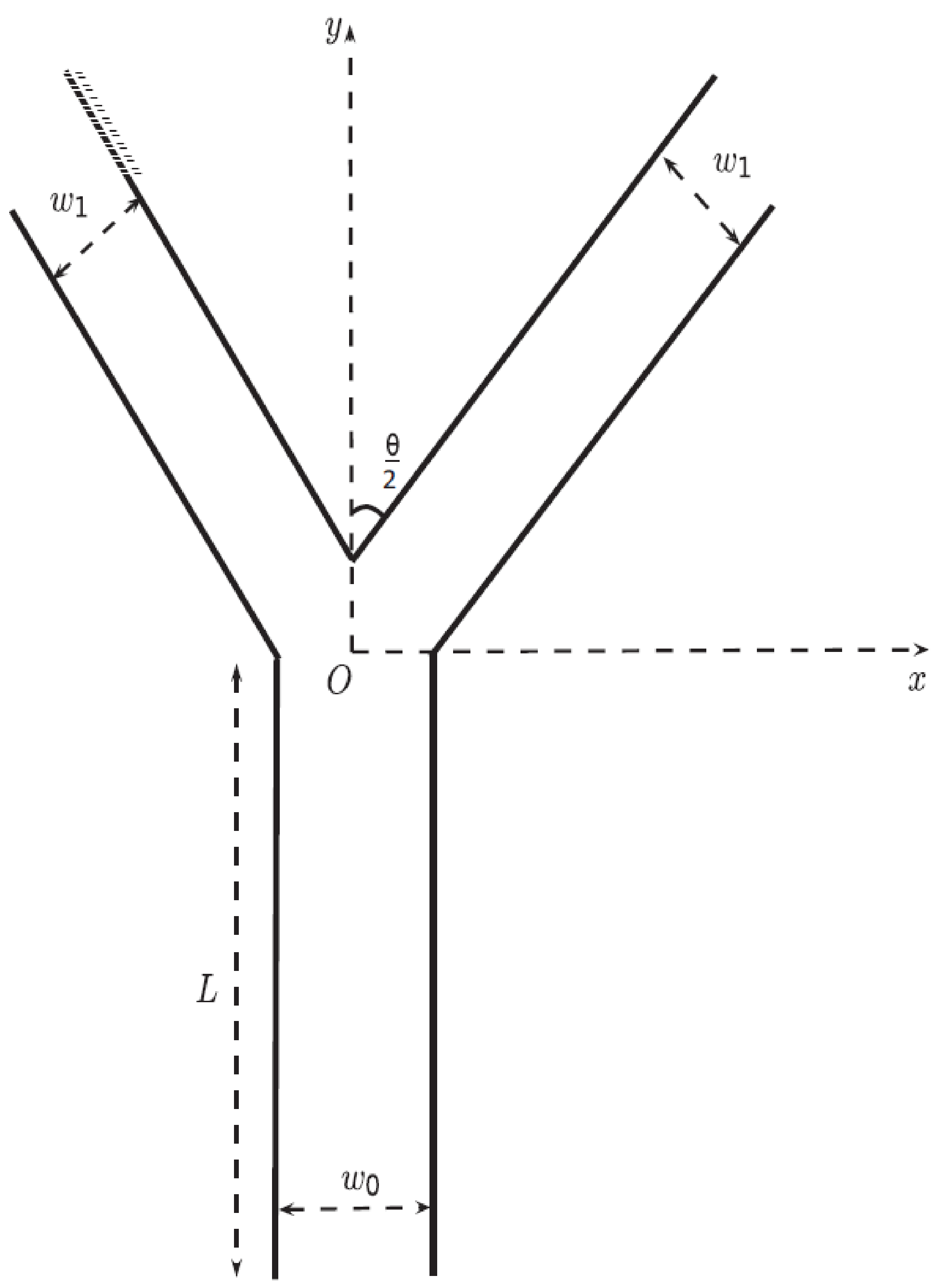
$$\varphi_{tt} - \Delta\varphi + \sin\varphi = 0, \tag{1}$$

on a bounded domain $\Omega \subset \mathbb{R}^2$ with Neuman boundary conditions,

$$\boldsymbol{\nabla}\varphi \cdot \mathbf{n} = 0,$$

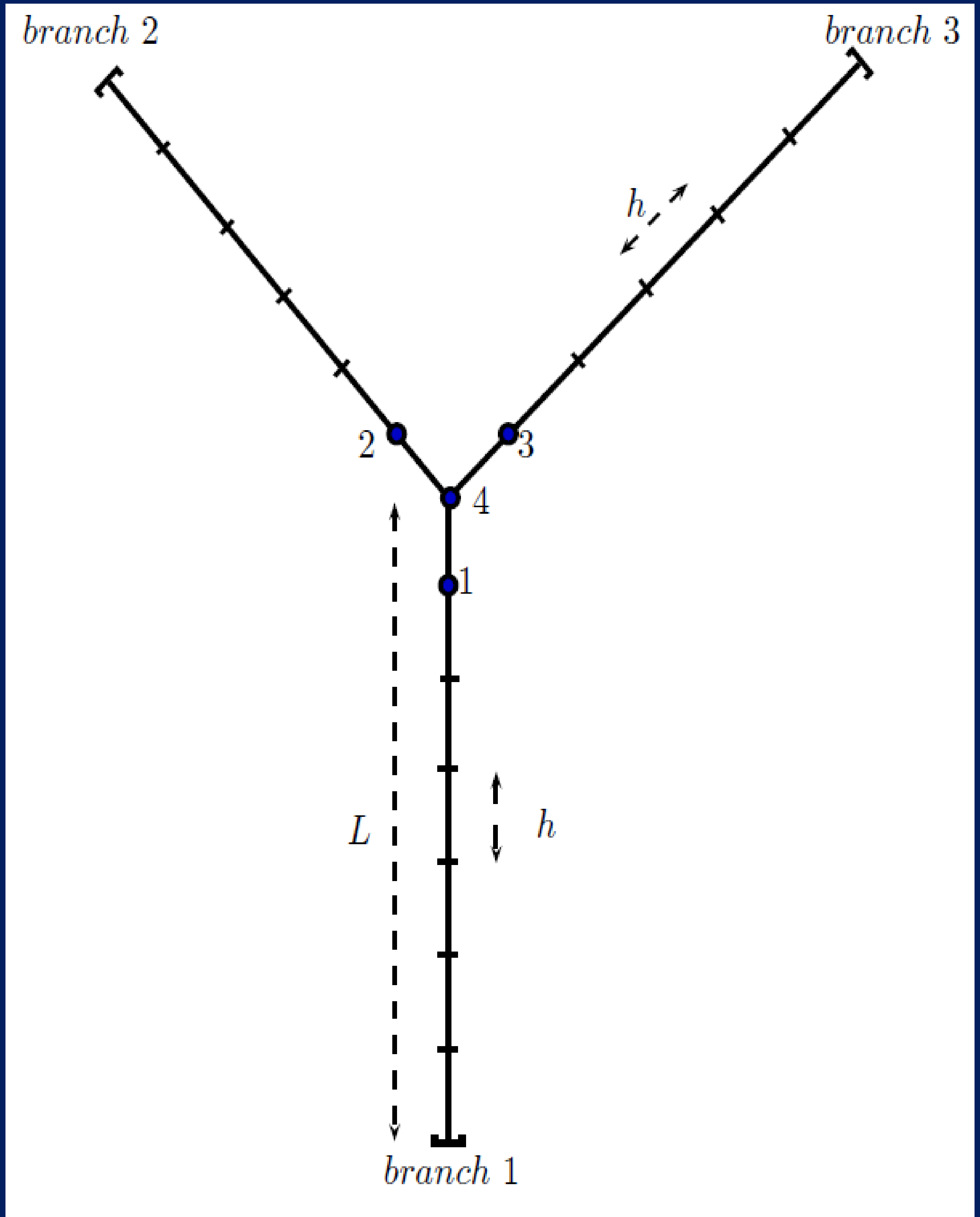
where $\mathbf{n}$ is an exterior normal. The t subscript indicates the time derivative and Δ is the usual Laplacian in spatial coordinates. This equation conserves the energy:

$$\mathcal{E} = \int_\Omega \left[\frac{1}{2}\varphi_t^2 + \frac{1}{2}|\nabla\varphi|^2 + (1 - \cos\varphi)\right] dx\, dy. \tag{2}$$



GOVERNING EQUATIONS (1D):

$$\varphi^i_{tt} - \varphi^i_{xx} + \sin\varphi^i = 0, \quad i = 1,2,3$$



Continuity argument:

$$\varphi^1(x = l) = \varphi^2(x = 0) = \varphi^3(x = 0)$$

At discrete level?

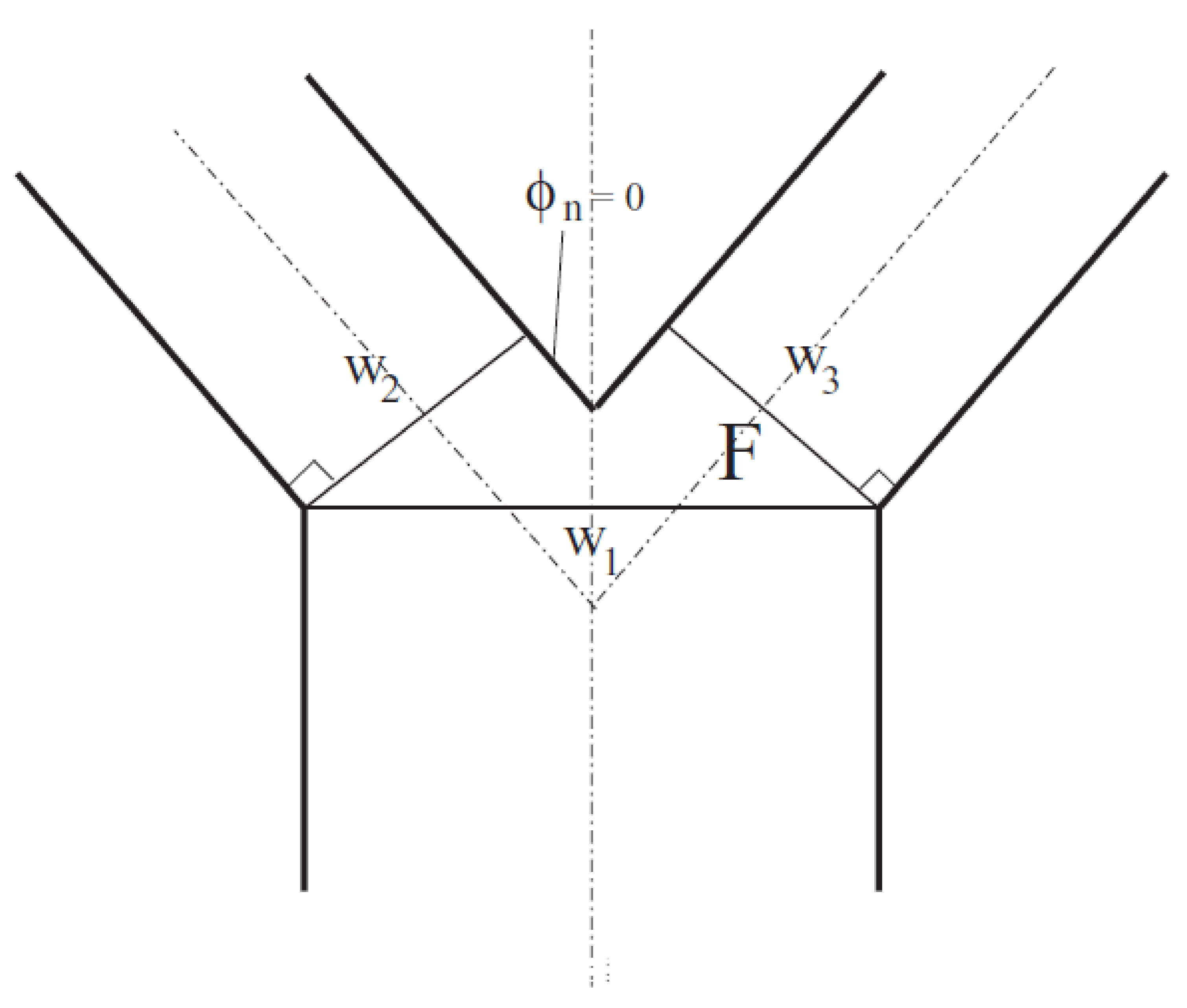
Derivation of "Kirchoff" law:

$$\int_F (\varphi_{tt} + \sin\varphi)\,dx\,dy - \int_{\partial F} \nabla\varphi \cdot \mathbf{n}\,ds = 0,$$

$$-w_1\varphi_x^1 + w_2\varphi_x^2 + w_3\varphi_x^3 = 0$$

$$-w_1(\varphi_4 - \varphi_1) + w_2(\varphi_2 - \varphi_4) + w_3(\varphi_3 - \varphi_4) = 0$$

$$\varphi_4 = \frac{w_1\varphi_1 + w_2\varphi_2 + w_3\varphi_3}{w_1 + w_2 + w_3}$$



Numerical methods:

$$\frac{\varphi_j^{n+1} + \varphi_j^{n-1} - 2\varphi_j^n}{\Delta t^2} - \frac{\varphi_{j+1}^n + \varphi_{j-1}^n - 2\varphi_j^n}{\Delta x^2} + \sin \varphi_j^n = 0,$$

Centered finite differences

$$\frac{1}{\Delta t^2}(\varphi^{n+1} - 2\varphi^n + \varphi^{n-1}, \psi) + \frac{1}{2}[\nabla(\varphi^{n+1} + \varphi^n), \nabla \psi] + (\sin \varphi^n, \psi) = 0,$$

P2 elements, triangular mesh, FreeFem++

ENERGY CONSERVATION IN FEM:

$$\mathcal{E}_n = \frac{1}{2}\int\left[\left(\frac{\varphi^{n+1}-\varphi^{n-1}}{2\Delta t}\right)^2 + |\nabla\varphi^n|^2 - 2(1-\cos\varphi^n)\right]dx\,dy.$$

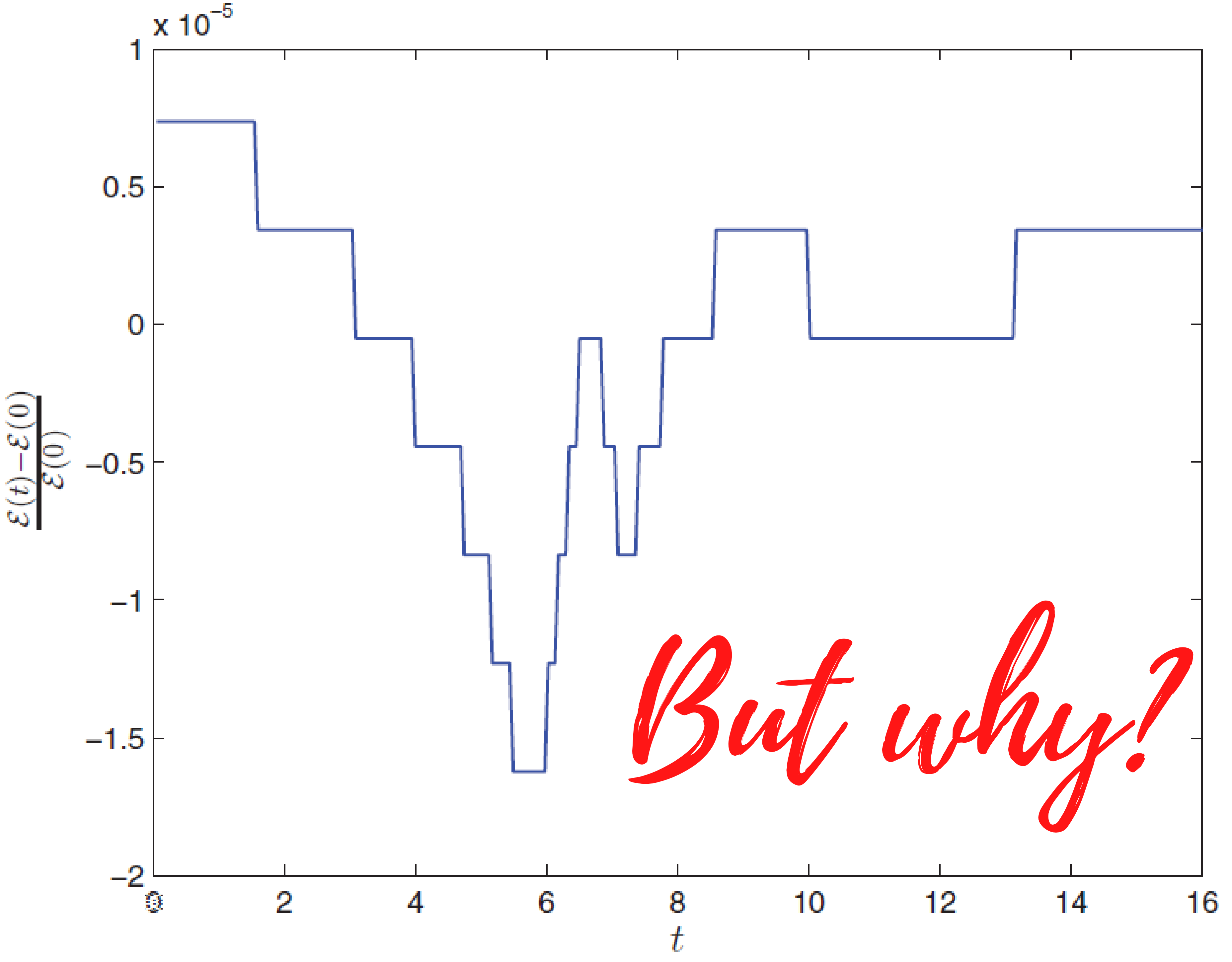


FIG. 4. (Color online) Relative energy $|\mathcal{E}_n - \langle\mathcal{E}_n\rangle|/\langle\mathcal{E}_n\rangle$ as a function of time for the 2D finite element solution of a breather propagating in a 2D domain.

Simulations

$$\gamma = \frac{1}{\sqrt{1-v^2}}.$$

Let us first consider the kink, its energy is

$$\mathcal{E}_k = 8\gamma.$$

ENERGY OF A KINK

In the sine-Gordon equation

$$w_1 \frac{8}{\sqrt{1-v_1^2}} = 2w_2 \frac{8}{\sqrt{1-v_2^2}}, \tag{13}$$

where we assume $w_2 = w_3$. This expression gives a critical velocity v_1 for which $v_2 = 0$:

$$v_k = \sqrt{1 - \left(\frac{w_1}{2w_2}\right)^2}. \tag{14}$$

CRITICAL VELOCITY:

Based on the energy conservation principle

CRITICAL VELOCITY ESTIMATION

BASED ON THEORY AND NUMERICAL SIMULATIONS

TABLE I. Kink critical velocities for the 2D model, the 1D effective model, and the energy estimate as a function of α. The widths of the branches are $w_1 = 1$ and $w_2 = w_3 = w_1 + \alpha$.

α	2D v_c	1D v_c	v_k From Eq. (14)
0.3	0.98	0.99	0.92
0.1	0.965	0.955	0.89
0	0.92	0.94	0.86
−0.1	0.885	0.85	0.83
−0.3	0.73	0.71	0.7

COMPARISON 1D/2D MODELS:

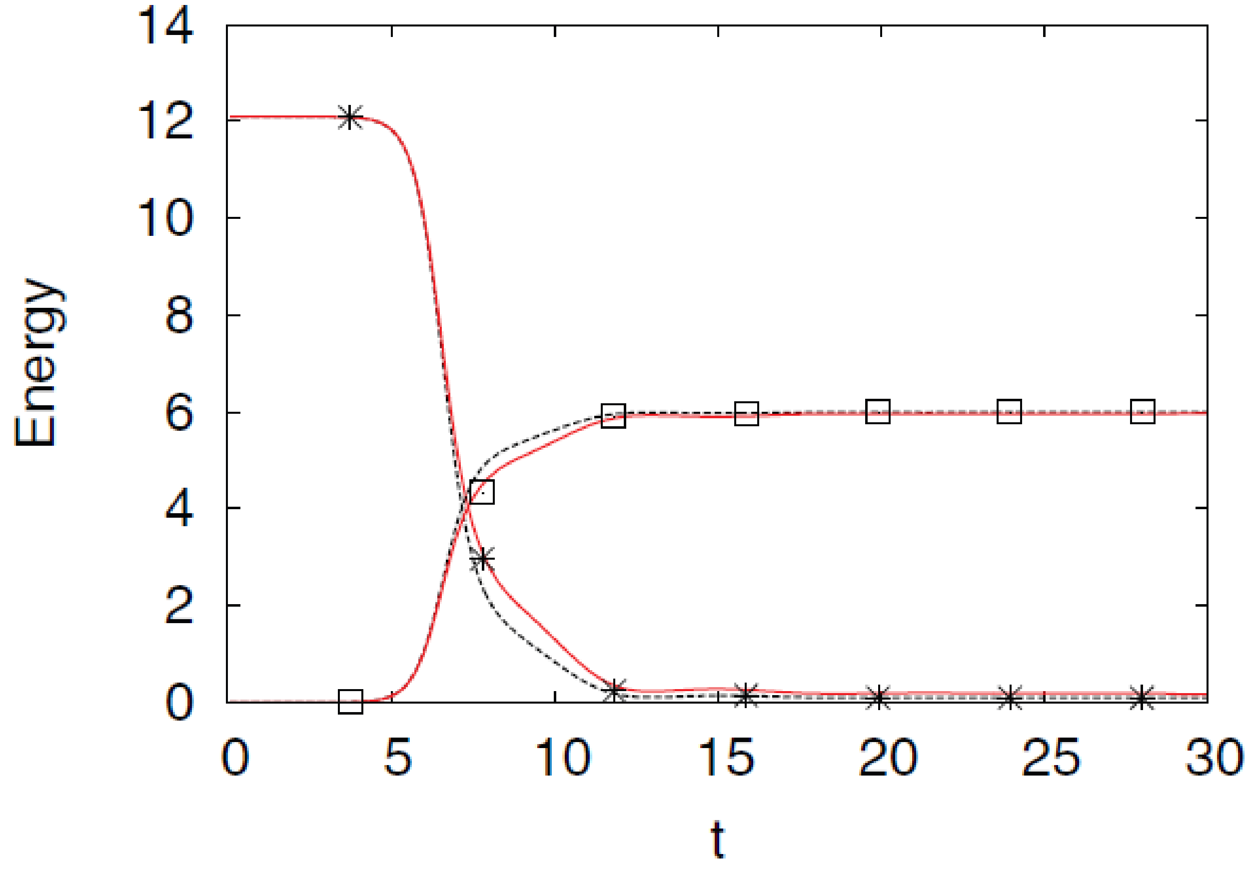


FIG. 7. (Color online) Time evolution of the energy for the kink motion in branches 1 and 2 for the T junction, shown as the solid line (red online), and for the Y junction, shown as the dashed line. The energy for the 1D effective model is plotted with points. The parameters are the same as those in Figs. 5 and 6.

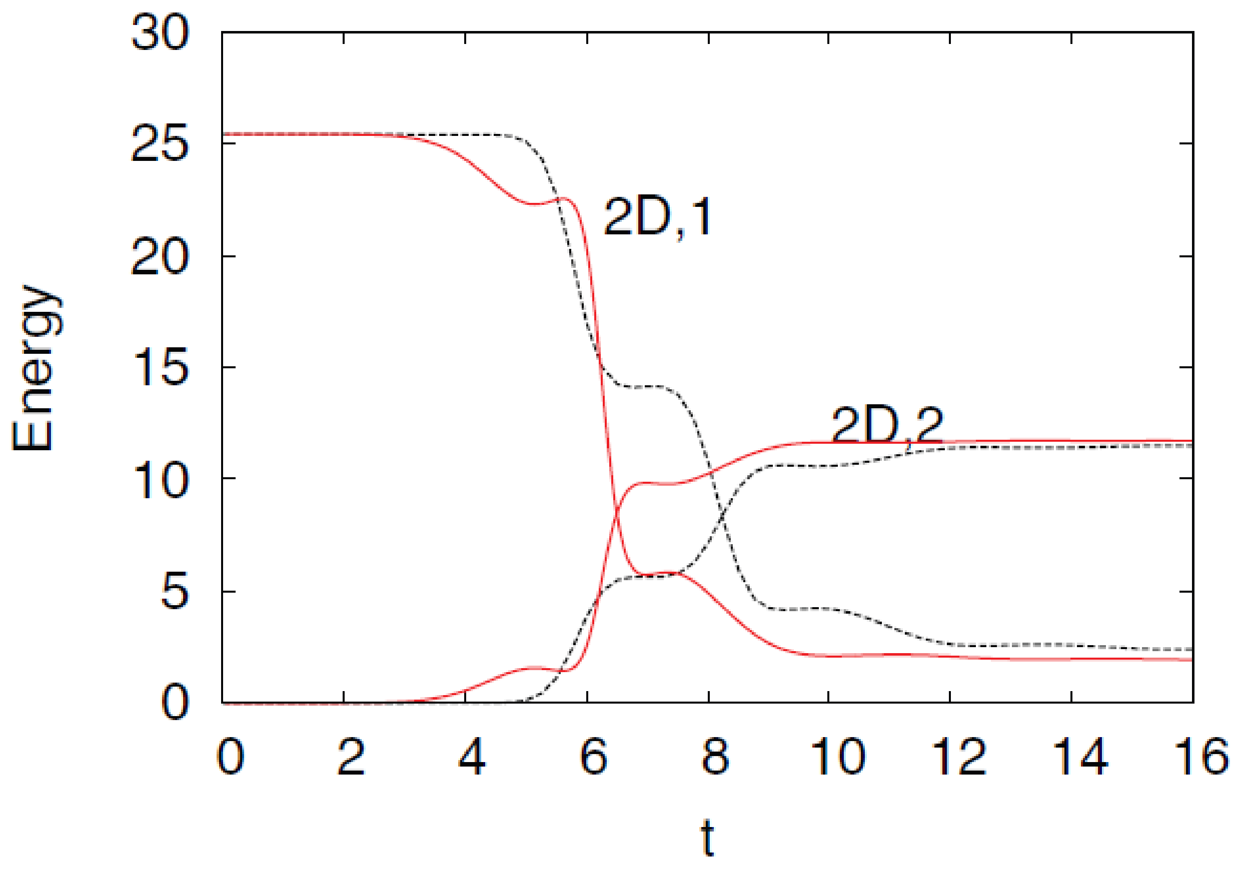


FIG. 10. (Color online) Time evolution of the energy in branches 1 and 2 for a breather for the 2D partial differential equation, shown as the solid line (red online), and the 1D effective model, shown as the dashed line. The parameters are $w_1 = w_2 = w_3 = 1$, $v_1 = 0.8$, $\omega_1 = 0.3$, and $x_0 = 10$.

BREATHER

COMPARISON WITH ANALYTICAL SOLUTION

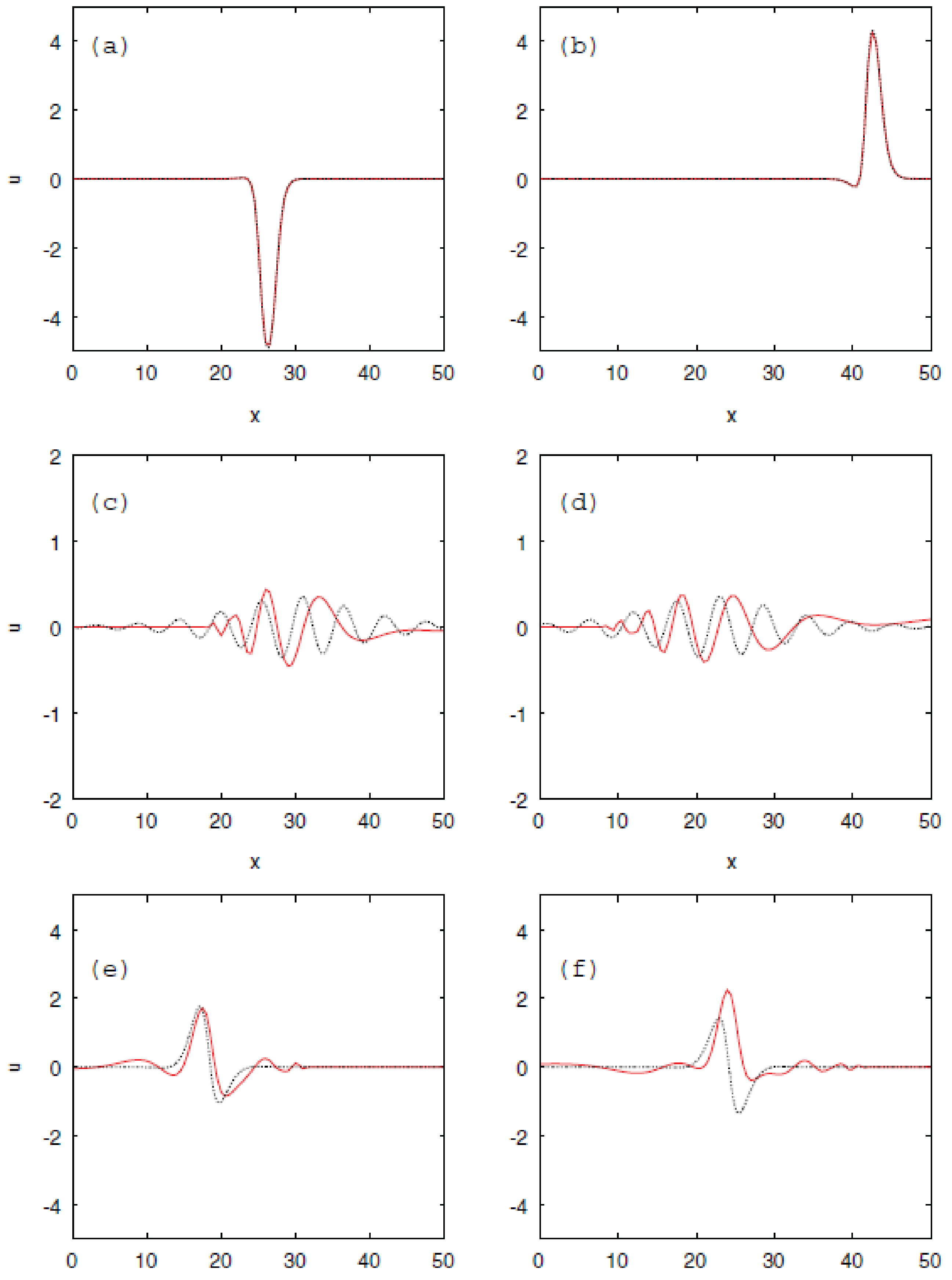


FIG. 12. (Color online) Snapshots of the breather analytical solution (dashed line) together with the numerical solution (continuous line), in branch 1 before the collision [panels (a) and (b)], in branch 1 after the collision [panels (c) and (d)], and in branch 2 [panels (e) and (f)]. The corresponding times are $t = 20.2, 40.4, 80.8, 90.9$ for panels (a)–(d) and $t = 80.8, 90.9$ for panels (e) and (f).

THE NEXT STEP:

symmetry

MDPI

Article

Coupling Conditions for Water Waves at Forks

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check for updates

Simulations

GOOD NEWS:

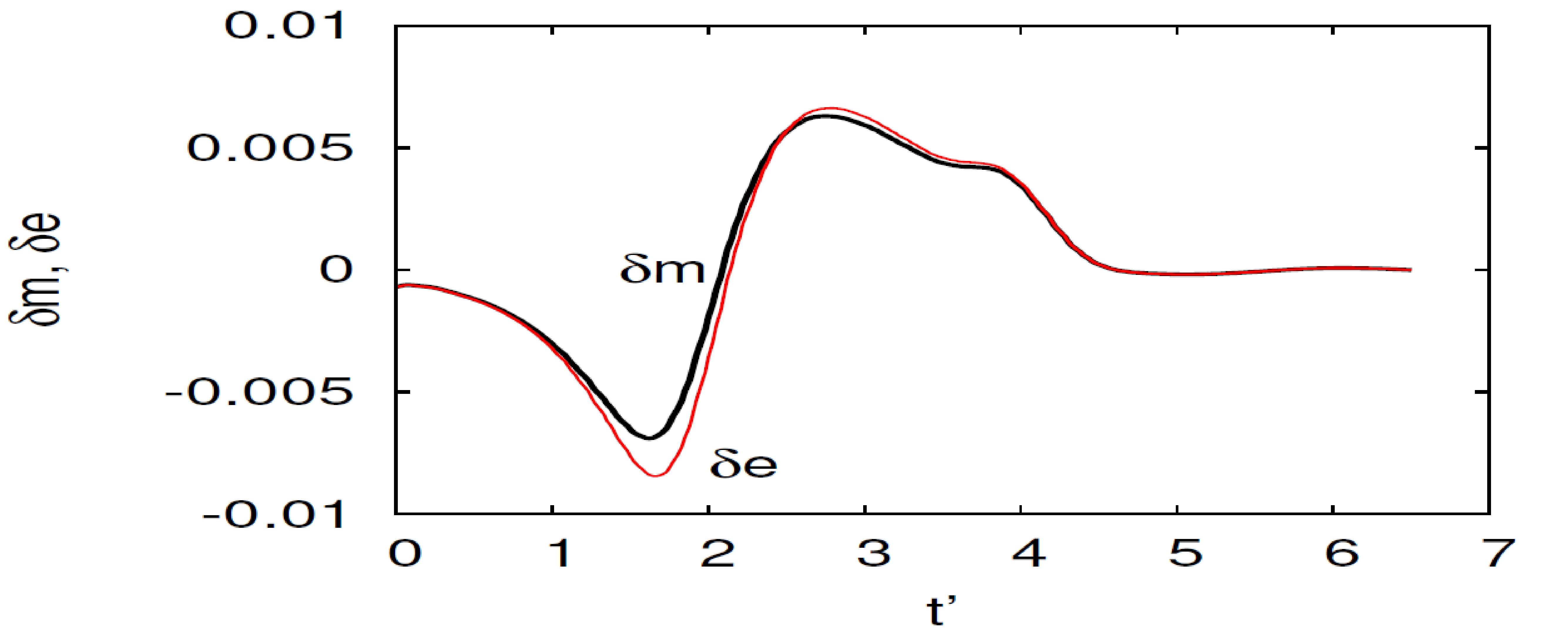

Figure 12. Time evolution of the mass and energy quantities δm (black online) and δe (red online) for $a/d = 0.1$.

BAD NEWS:

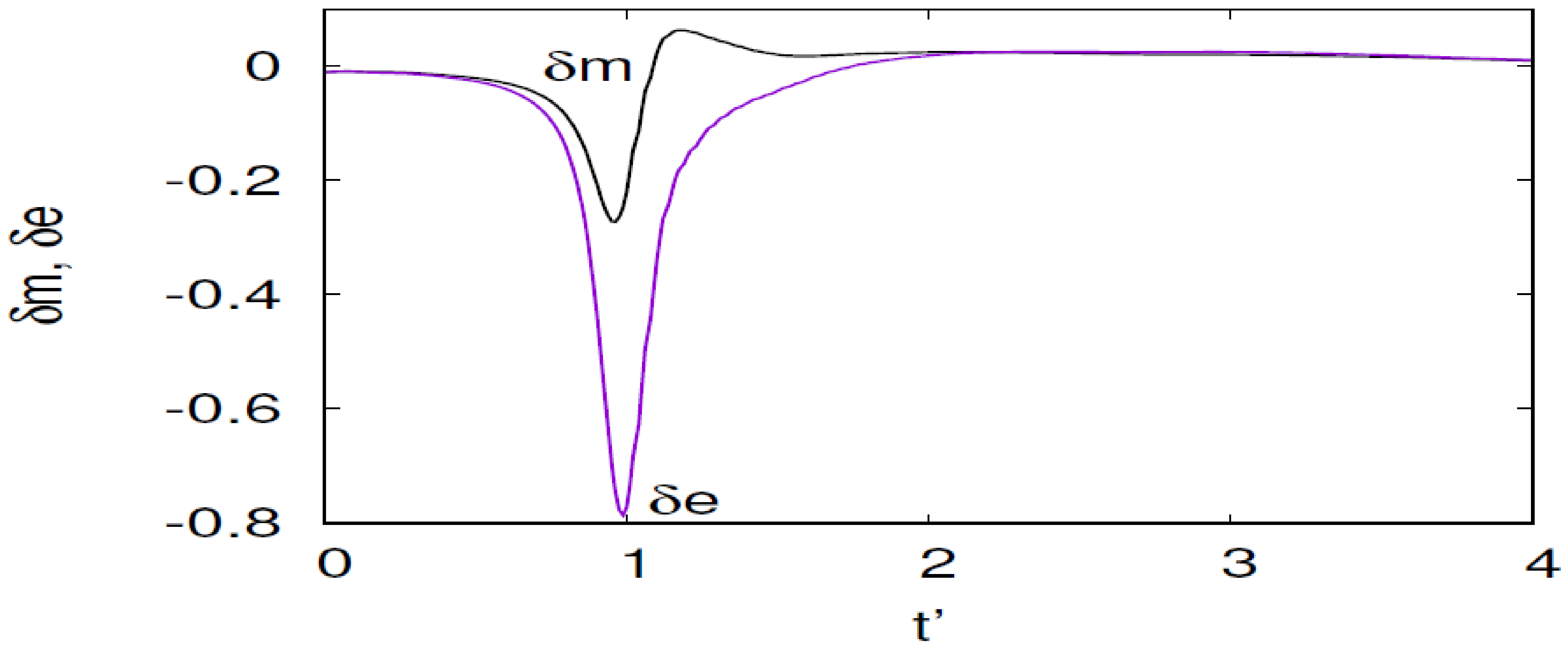

Figure 15. Time evolution of the mass and energy quantities δm (black online) and δe (purple online) for $a/d = 2$.

Possible reasons:

VECTORIAL

NSWE are multi-component comparing to the sine-Gordon

NON-INTEGRABLE

No Lax pair exists for NSWE. Thus, only a finite number of conserved current

DISCONTINUOUS SOLUTIONS

Solutions will evolve in general into a shock-wave in finite time

NON-UNICITY (IN 2D)

In certain situations the weak solutions are known to be non-unique in NSWE and in isentropic Euler equations

A FEW WORDS ON THE LAST POINT:

It is also well-known that hyperbolic systems of conservation laws develop discontinuities in finite time even if we start from smooth initial data [59]. In water wave theory they are known as *hydraulic jumps* (or undular bores*) [93]. This phenomenon is known as the *gradient catastrophe* or *breakdown of classical solutions*. This obstacle was overcome by introducing the so-called *weak* solutions. Unfortunately, weak solutions fail to be unique. The help comes from Physics, namely from the second law of Thermodynamics. One can stipulate that admissible solutions satisfy some additional *entropy inequalities*. The quest for well-posedness theory of the Cauchy problem for hyperbolic conservation laws is more than one century old. Unfortunately, it was shown recently in [14] that entropy conditions do not single out unique weak solutions in 2D even under very strong assumptions on the initial data $(\rho_0, \boldsymbol{v}_0) \in W^{1,\infty}(\mathbb{R}^2)$ (here $|\infty| = \aleph_0$) [16]:

Theorem 1. *There are* Lipschitz *continuous initial data* $(\rho_0, \boldsymbol{v}_0)$ *for which there are infinitely many bounded admissible solutions* $(\rho, \boldsymbol{v})$ *to System* (2.72), (2.73) *on* $\mathbb{R}^2 \times \mathbb{R}_0^+$ *with* $\inf \rho > 0$. *These solutions are locally* Lipschitz *on a finite interval on which they all coincide with the unique classical solution.*

The solutions described in the last Theorem were called *non-standard solutions* in [15].

Some awkward questions:

- How to define weak solutions in 2D (and 3D, ...) to recover their unicity?
- What do we compute in our multi-D simulations?
- What about other nonlinear PDEs?

We don't have to answer today!

Thank you very much for your attention!

THANK YOU VERY MUCH, JEAN-GUY!

If you have any further questions:

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