ONL2022 : Nonlinear waves and networks
60th Birthday of Jean-Guy Caputo
Rouen July 4,5 2022

PROGRAM and ABSTRACTS:

Monday July 4th

9:30 10:00 Welcome of the participants and opening remarks

Mathematical biology : chairman Jacques Demongeot

10:00 10:45 Gustavo Cruz
Approximations and estimates of the Covid 19 pandemic in Mexico

10:45 11:30 Jacques Demongeot
Turing structures, 0-diffusion sets and applications in morphogenesis

11:30 12:15 Benoit Sarels
Interaction of clines under the prism of generalized travelling waves

12:15 14:00 ————LUNCH BREAK————

Nonlinear waves : chairman Benoit Sarels

14:00 14:45 Denys Dutykh
Nonlinear waves in Y-junctions

14:45 15:30 Mads Peter Sorensen
Solitons and coherent structures in optics and superconductivity

15:30 16:00 ————BREAK————

16:00 16:45 Efstathios Charalampidis
Recent Advances on Localized Solutions in NLS systems: Theory and Computation

16:45 17:30 Ildar Gabitov
Nonlinearity and High Speed Communications: new view on the old problem

20:00 ——————SPECIAL EVENT———————
Conference dinner, Hotel Radisson in front of Rouen train station

Tuesday July 5th

Graph theory and networks, chairman Arnaud Knippel

9:30 10:15 Irene Sciriha
When networks have the same canonical double cover

10:15 11:00 Imene Khames
Nonlinear normal modes in a network with cubic couplings

11:00 12:15 ——————POSTER SESSION——————

12:15 14:00 ————LUNCH BREAK———————

Condensed matter physics, chairwoman Irene Sciriha

14:00 14:45 Regis Melin
Recent results on multiterminal Josephson junctions

Turing structures, 0-diffusion sets and applications in morphogenesis

J. Demongeot
UGA & IUF

Mathematical models in morphogenesis obey multiple constraints concerning notably: i) the cell proliferation depending on the local curvature of the contour lines of cell density, such as it favors the amplification of the tissue concavities (like in the gastrulation process), ii) the auto-assemblage of biological components (mainly the effectors of the expression of the proteins needed in the final assembled structure) on the 0-diffusion sets of Turing periodic (in space) structures and (iii) the genetic expression which contributes to build morphogenetic proteins in the framework of genetic regulatory networks whose dynamics presents multiple attractors. These three dynamical processes are coexisting and the final form observed at the end of the morphogenetic process results from their interaction, as the direct consequence of the nonlinearities of the differential equations modelling their dynamics and of the existence of circuits in their Jacobian interaction graph.

Two applications of these multiple and complementary mathematical approaches to biological morphogenetic systems are presented: gastrulation and feather genesis.

References

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M. Rachdi, J. Waku, H. Hazgui & J. Demongeot
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Denys Dutykh
Laboratoire LAMA and CNRS
University of Chambery

Nonlinear waves in Y-junctions

In this talk, I would like to present a couple of works we did together in close collaboration with Jean-Guy Caputo on nonlinear wave propagation across a special Y-junction geometry. All merits of this work are due to Jean-Guy and all imperfections are due to the speaker.

Regis Melin
Universite Grenoble Alpes and CNRS

Recent results on multiterminal Josephson junctions

A supercurrent flows between two superconductors separated by a thin insulating oxide layer, which is known as the DC-Josephson effect. Pairs of opposite-spin electrons (i.e. Cooper pairs) are transferred between both sides of the contact. Recently, contacts between three or more superconductors were investigated. Considering a junction between three superconductors, the device can be biased with two independent voltage differences, and with two independent phase differences. Then, if the bias voltages are opposite, a novel mechanism appears: the so-called quartets [1], which involve exchange between the partners of two Cooper pairs. Then, at finite bias voltage, interesting connections can be established with the theory of quantum systems with time-periodic Hamiltonians, known as the Floquet theory [2,3].

In this talk, I will focus on a recent four-terminal experiment realized in the Harvard group [4], starting with the simple picture emerging from perturbation theory in the tunneling amplitudes [5]. Next, I will address the Floquet dynamics for a single quantum dot coupled to four superconducting leads [6], and double quantum dot coupled to four superconducting leads and to normal leads [7]. The single and double quantum dot calculations can be seen as a way to simplify the geometry into finite elements, taking into account the interplay between the Floquet resonances and relaxation. Those theoretical model calculations provide support to the experiment in Ref. [4].

[1] Production of non-local quartets and phase-sensitive entanglement in a superconducting beam splitter, A. Freyn, B. Douçot, D. Feinberg and R. M\'elin, Phys. Rev. Lett. 106, 257005 (2011).

[2] Simple Floquet-Wannier-Stark-Andreev viewpoint and emergence of low-energy scales in a voltage-biased three-terminal Josephson junction, R. Mélin, J.-G. Caputo, K. Yang and B. Douçot, Phys. Rev. B 95, 085415 (2017).

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[6] Inversion in a four terminal superconducting device on the quartet line: II. Quantum dot and Floquet theory, R. Mélin and B. Douçot, Phys. Rev. B 102, 245435 (2020).

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Recent Advances on Localized Solutions in NLS systems: Theory and Computation

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Abstract

The primary focus of this talk is on the exploration of the configuration space of solutions to the Nonlinear Schrödinger (NLS) equation, either single and continuous or discrete, e.g., Ablowitz-Ladik and DNLS as well as multi-component versions thereof. This will be accomplished by the introduction of algorithmic procedures, called continuation methods that are capable of tracing out branches of fixed points/roots to nonlinear equations as one (or more) of the free parameters of the underlying system is varied. As soon as branches of solutions are obtained, a spectral stability analysis of the pertinent waveforms will be applied in order to shed light on the stability trait of the solutions as well as potential bifurcations.

In the first part of the talk, we will consider one- and two-component NLS systems at the continuum limit in 2D and 3D. Alongside the commonly used sequential and pseudo-arclength continuation methods, we will present a novel and powerful continuation technique called the Deflated Continuation Method (DCM) which tries to find/construct undiscovered/disconnected branches of solutions by eliminating known branches. Indeed, upon the application of the DCM to the underlying NLS systems, we will present novel coherent structures that had not been reported before, and discuss bifurcations involving such states. The second part of the talk will be devoted to the so-called Salerno model which interpolates homotopically between the integrable Ablowitz-Ladik (AL) model and (non-integrable) discrete NLS (DNLS) equation. Through the use of fixed-point techniques and numerical continuation, we will study the existence, stability and dynamics of discrete Kuznetsov-Ma breathers therein. Novel findings will be reported and future directions of study will be discussed.

References

[1] N. Boullé, E.G. Charalampidis, P.E. Farrell and P.G. Kevrekidis, Deflation-based Identification of Nonlinear Excitations of the three-dimensional Gross-Pitaevskii equation, *Phys. Rev. A*, **102** (2020), 053307.

[2] E.G. Charalampidis, N. Boullé, P.E. Farrell and P.G. Kevrekidis, Bifurcation analysis of stationary solutions of two-dimensional coupled Gross-Pitaevskii equations using deflated continuation, *Commun. Nonlinear Sci. Numer. Simulat*, **87** (2020), 105255.

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[5] E.G. Charalampidis, J. Cuevas-Maraver, D.J. Frantzeskakis and P.G. Kevrekidis, *Rom. Rep. Phys.*, **70** (2018), 504.

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WHEN NETWORKS HAVE THE SAME CANONICAL DOUBLE COVER

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Abstract

The canonical double cover (CDC) of a base graph G is its direct product $G \times K_2$ with K_2. Its automorphism group contains a copy of $\mathrm{Aut}(G)\times\mathrm{Aut}\ (K_2)$. Unexpected symmetries may occur in $\mathrm{Aut}(G\times K_2)$ that are not a lifting of $\mathrm{Aut}(G)$, giving rise to multiple non–isomorphic base graphs. The inverse problem of determining the base graphs which are pre–images of a CDC is hard, requiring various techniques. Non–isomorphic graphs with isomorphic CDCs have the same degree sequence. They also have the same number of walks of arbitrary length from corresponding vertices. We define a walk–colouring, partitioning the vertices of a graph into colour classes, by assigning the same colour to vertices having the same number of walks of any specific length starting from them. It is known that graphs have isomorphic CDCs if and only if their 0–1–adjacency matrices are permutationally congruent. We show that the graphs with isomorphic CDCs are related by Ryser–switching of disjoint edges with the same end-vertex colouring.

*Joint work with Luke Collins.

Solitons and coherent structures in optics and superconductivity

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Abstract

In this presentation the dynamics of coherent structures as solitons, solitary waves and vortices will be exemplified by cases from nonlinear fiber optics and superconductivity. The examples will in part pertain to cooperation between LMI-INSA, University of Rouen, and The Technical University of Denmark.

For fiber optics we have extended the energy method for the perturbed nonlinear Schrödinger (NLS) equation, where one governing equation for a slowly varying parameter in a soliton solution is determined. We have extended this idea by deriving a Lagrange function for multiple slowly varying parameters in a soliton solution by variation of the Lagrangian and by invoking generalized forces from the perturbations. From this method we can obtain a system of nonlinear ordinary differential equations for arbitrary numbers of slowly varying parameters. The method is exemplified by a perturbed NLS equation modelling an optical laser fiber.

Vortices in type II superconductors are another example of strongly coherent patterns. The vortices are modelled by the Ginzburg–Landau equations governing a quantum mechanical coherent variable coupled to the magnetic field. We have investigated the dynamics of vortices in a superconductor with pinning sites. Close to the first critical magnetic field separating the perfect diamagnetic state from the state with penetrating vortices, we observe complex dynamics of the vortices and their interaction with the pinning sites. In particular the simulation results demonstrate hopping of vortices between pinning sites, influenced by external magnetic fields and external currents. Future work pertain to studies of using type II superconductors with artificial pinning sites as permanent magnets. Vortices are introduced into the superconductor by an external magnetic field. Removing the magnetic field leaves back the pinned vortices on impurities creating a permanent superconducting magnet without an external magnetic field.